Overlooked Phenomena in Reflex Oscillators

Amit K Varshney, B N Biswas, Arindum Mukherjee

Dept. of Electronics and Communication Engineering, SKFGI, India
CIT, Kokrajhar, India

Abstract

Analysis has been carried out to obtain the expression of beam current of Reflex Klystron Oscillator (RKO). The beam current has been found to contain both in phase and quadrature phase component which was overlooked by the earlier workers. These affect both the amplitude and frequency of the oscillator. Effects of delay due to various modes have been demonstrated for the first time. Effect of synchronization on the phase locking characteristics of the oscillator has also been demonstrated.

Keywords

In Phase Component; Modulation Index; Quadrature Phase Component; Synchronizing Current

I. Introduction

The RKO finds useful applications in double-detection receivers or as a frequency modulated oscillator in low power transmitters [1]. In the development of the theoretical basis of the Klystron operation, kinematics was used by the W C Hahn neglecting the space charge effects. Latter W C Hahn hinted how space charge effect can be taken into account to predict the tube behavior, On the other hand, latter Simon Ramo developed the theory of velocity modulation by incorporating displacement current and variations of fields charge and current densities with beam cross section, length and time by invoking Maxwell’s equations. He explained the principle of velocity modulation through the existence of two slow space charge waves – one of the waves propagates with a velocity slightly greater and the other slightly less than that of the beam current [2]. In fact this concept helped Webster to explain the phenomenon of de-bunching in Klystron. This creates difficulty in realizing the ideal efficiency. According to him de-bunching does not limit the number of bunches a beam may contain but limits length. But the author feels that the final conclusions more or less agree with the outcome of Kinematic principle of velocity modulation [3-10].

RKO is a velocity modulated tube and is fairly an old topic. As such a large number of papers have been published on RKO.

II. Fourier Component of Beam Current

If we plot the relation between the arrival and departure time of electrons at the cavity grids (Fig. 2), we will observe that for X<1, the electrons which arrived during the time interval ‘1’ left the resonator grids during the time interval ‘3’, whereas for X≥1, the electrons which arrived during the time interval ‘2’ left the resonator grids during the time interval ‘4’, ‘5’ and ‘6’. As a result of the principle of charge conservation we obtain two different relations, namely [11].

\[
\begin{align*}
\frac{dI(t)}{dt} &= I_0 dt_0, \text{for } X < 1, \tag{1} \\
\frac{dI(t)}{dt} &= I_0 \sum dt_0, \text{for } X \geq 1 \tag{2}
\end{align*}
\]

The summation has to be taken over all the three intervals. In these relation \( \frac{dI(t)}{dt} \) is the returning beam current and \( I_0 \) is the forward beam current. Obviously the returning beam current in both the cases is periodic in return and so it can be expressed by Fourier series as

\[
\frac{dI(t)}{dt} = a_0 + \sum a_n \cos n(\theta) + \sum b_n \sin n(\theta) \tag{3}
\]

where \( \theta = \omega t - \omega T_0 \), \( t \) being the arrival time of electrons at the cavity grids and \( T_0 \) the dc transit time in the repeller space. Let \( \varphi = \omega T_0 = \omega NT_1 \), \( 2\pi N \), \( T_1 \) being the time period of the

Fig. 2: Plot Showing the Relation Between Arrival and Departure Time of Electron Beam to the Resonator for Different Values of Bunching Parameter X.
signal and N, the mode number, is given by N=(n-1/4), n=1, 2,....

The transit time in the repeller space can be easily derived as

\[ T = T_0 \sqrt{1 + \frac{V\beta}{V_0} \sin \omega t_0} \]  

(4)

where \( \beta \) is the beam coupling coefficient.

Let \( \alpha = \frac{V}{V_0} \), \( V \) being the steady state amplitude of rf signal and \( V_0 \) the dc beam voltage.

Thus, from (4)

\[ T = T_0 \sqrt{1 + \alpha\beta \sin(\omega t_0)} \]

(5)

Let \( \alpha\beta = \frac{2X}{\psi} \)

(6)

where \( X \) is the bunching parameter or the modulation index.

Now we have \( t = t_0 + T \), or, \( \omega t = \omega t_0 + \omega T \)

Where \( t_0 \) is the departure time of electrons

\[ \omega t = \omega t_0 + \omega T_0 \sqrt{1 + \frac{V\beta}{V_0} \sin \omega t_0} \]

(7)

By simple algebraic manipulation and taking the help of (6) it can be shown that

\[ \theta = \theta_0 + \psi \sqrt{1 + \frac{2X}{\psi} \sin \theta_0} - \psi \]

(8)

In (3) \( \alpha t_0 \) turns out to be the dc beam current \( I_0 \) and

\[ a_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} i(t) \cos n(\theta)d\theta \]

(9)

\[ b_n = \frac{1}{\pi} \int_{-\pi}^{+\pi} i(t) \sin n(\theta)d\theta \]

(10)

The fundamental component of beam current is given by

\[ a_1 = \frac{1}{\pi} \int_{-\pi}^{+\pi} i(t) \cos(\theta)d\theta \]

(11)

\[ b_1 = \frac{1}{\pi} \int_{-\pi}^{+\pi} i(t) \sin(\theta)d\theta \]

(12)

Substituting \( \theta \) from (8) into (11) and (12) and remembering that

\[ \sum_{i=2}^{n} \left( \frac{I_0}{dt} \right)_i = \sum_{i=2}^{n} \frac{I_0}{1 + \frac{X \cos \omega t_0}{\sqrt{1 + \frac{2X}{\psi} \sin \omega t_0}}} \]

(13)

and, \( d\theta = 1 + \frac{X \cos \theta_0}{1 + \frac{2X}{\psi} \sin \theta_0} \) d\( \theta_0 \),

(14)

one obtains

\[ a_1 = \frac{I_0}{\pi} \int_{\Theta_0}^{\Theta} \cos \left( \theta_0 + 2\pi N \sqrt{1 + \frac{2X \sin a}{2\pi N} - 2\pi N} \right) d\theta_0 \]

And \[ b_1 = \frac{I_0}{\pi} \int_{\Theta_0}^{\Theta} \sin \left( \theta_0 + 2\pi N \sqrt{1 + \frac{2X \sin b}{2\pi N} - 2\pi N} \right) d\theta_0 \]

where \( a \) and \( b \) is obtained by evaluating

\[ -\pi = a + 2\pi N \sqrt{1 + \frac{2X \sin a}{2\pi N} - 2\pi N} \]  

(15)

\[ \pi = b + 2\pi N \sqrt{1 + \frac{2X \sin b}{2\pi N} - 2\pi N} \]

(16)

It is interesting to note that the beauty of the transformation is that it not only avoids the singularity problem but also absorbs the summation sign.

From the equivalent circuit of reflex klystron (Fig. 3) we can write

\[ \frac{dX(t)}{dt} + \frac{1}{L} \int_{t_0}^{t} [v(t)dtler + l_b(t)l_s(t)\psi(t)dt] = i_s \]

(17)

where \( l_b(t) \) and \( l_s(t) \) are the in-phase and quadrature phase component of beam current which also takes delay into account, and \( i_s \) is the synchronizing current.

For simplicity let us consider \( x = p\psi \), \( p = \pi\beta N/V_0 \)

where \( v = v(t)\exp(j\psi(t)) \) and \( x = X(t)\exp(j\psi(t)) \).

Since (17) is non linear equation, we need to convert into an equivalent linear one by applying linear describing function technique. By this technique we can write,

\[ i_b(X) = i_b(X) + j_i_b(x) = -i_b(\alpha_1 x + \alpha_2 x^3 - \alpha_3 x^5)
\]

(18)

\[ -j_i_b(\beta_1 X + \beta_2 x^3 - \beta_3 x^5) \]

where \( \alpha_1, \alpha_2, \alpha_3, \beta_1, \beta_2, \) and \( \beta_3 \) are the polynomial coefficients.

Considering

\[ i_s = I_s \exp(j\omega_l(t)) \]

(19)

\[ \psi(t) = \omega_l(t) + \theta(t) \]

(20)

Phase difference between input and output is given by

\[ \theta(t) = \omega_l(t) - \omega_l(t) - \theta(t) \]

(21)

Further we can write

\[ j\omega = \frac{1}{X(t)} \frac{dX(t)}{dt} + j\omega_l + j \frac{1}{X(t)} \frac{dX(t)}{dt} \]

(22)

So, (19) may be written as

\[ i_s = I_s \exp(-j\theta) \exp(j\psi) \]

(23)
From (17) we can write
\[ i_r = C \frac{dv}{dt} + \frac{1}{L} \int vdt + Gv \]  
(24)
We can write (24) in terms of \( X \) as
\[ i_r = C \frac{dx}{dt} + \frac{1}{L} \int xdt + Gx \]
\[ i_r = (j \omega C + \frac{1}{j \omega L}) x \]
(25)
From (22) we can write (25) as
\[ i_r = [2C \left( \frac{1}{X} \frac{dX}{dt} + j \omega x + \frac{1}{j \omega} \frac{d\theta}{dt} - j \omega x \right) + G] X \]
(26)
Inserting (26), (23) and (18) in (17) and equating real and imaginary parts and also considering delay in the synchronizing current we obtain
\[ \frac{dX(t)}{dt} = \frac{I_0}{2C} \left[ \alpha X(t) - \frac{C}{\omega L} + \frac{3}{4} \alpha x X^3(t) + \frac{\omega_0}{2Q} E_x \cos \theta(t) \right] \]
(27)
\[ \frac{d\theta(t)}{dt} = \Omega + \frac{I_0}{2C} \left[ \beta x X \left( t - \frac{3}{4} \beta_1 X^3(t) \right) - \frac{\omega_0}{2Q} \beta_1 X^3(t) \sin \theta(t) \right] \]
(28)
where \( I_0 / 2C \) has been replaced by \( (\omega_0 E_x) / (2Q) \) and \( \Omega = \omega_0 - \omega_1 \).

III. Results and Discussion

The effect of delay causes the amplitude \( X(t) \) to attain steady state value at a later time (Fig. 4(a) and 8(a)), the effect being more pronounced at higher mode \( (n=2, \) Fig. 5(a)). The presence of quadrature component causes change in frequency \( d\theta / dt \) (Fig. 4(b) and 5(b)) and hence phase \( \theta \) (Fig. 6). In the presence of synchronizing signal, the quadrature component stabilizes the amplitude earlier (Fig. 6a) compared to that in the absence of the same. The effect on frequency change and phase change is shown in Fig. 7(b) and 7(c) respectively. The presence of delay (in the presence of synchronizing signal) locks the frequency and phase of the RKO at a later time (Fig. 8(b) and 8(c)). All quantities are normalized with respect to \( I_0 / 2C \).

Fig. 4: Variation of amplitude (a) and frequency change (b) with time for the mode \( n=1 \) in the absence of synchronizing signal.

Fig. 5: Variation of amplitude (a) and frequency change (b) with time for the mode \( n=2 \) in the absence of synchronizing signal.

Fig. 6: Effect on the phase change of the signal due to the presence of quadrature component that also causes change in frequency in the absence of synchronizing signal (mode \( n=1 \)). Amplitude change is ignored here.
IV. Conclusion

It is thus concluded that when the modulation index is equal to and greater than one, the relation between the departure and arrival time of electrons at the cavity grids is a multivalued function resulting in the appearance of quadrature terms in the Fourier component of beam current. As a result of this during the growth of oscillations, both amplitude and frequency changes till the steady state is reached. Delay due to modes has a marked effect on amplitude and frequency response even in the presence of synchronizing signal. Moreover, presence of quadrature components produces asymmetry in the frequency response of a synchronized oscillator. The details will be presented in the accompanying paper.

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References


Amit Kumar Varshney, an M Tech in Electronics and Communication Engineering (Microwave) Gold Medalist securing First Rank from Burdwan University is working as Assistant Professor at Supreme Knowledge Foundation Group of Institutions, Mankundu, Maulana Abul Kalam Azad University of Technology, West Bengal, India. He is a budding researcher in the area of microwave engineering, in general, and vacuum electron devices (microwave tubes), in particular. Working towards his Ph D degree, he has published himself in a good number of peer-reviewed journals and conference proceedings. He has successfully collaborated for his research with the national laboratories like Central Electronics Engineering Research Institute (CEERI), Pilani, India (CSIR) and Microwave Tube Research and Development Centre (MTRDC), Bengaluru, India (DRDO) as evidenced by his co-authorship of several research papers with the scientists of these laboratories (CSIR-CEERI and DRDO-MTRDC).

He has developed the analysis of helical slow-wave structures with tapered-geometry dielectric helix supports for wideband electronic warfare traveling-wave tubes (TWTs). He has also developed the concepts for research in the area of klystrons. He is now exploring the potential of metamaterial loading of the helix of a TWT and the cavity of a klystron for potential of such loading for futuristic application in improving the performances of these devices.