Analysis of Heart Beat Signals using Hilbert and Hilbert-Huang Transform

Mehak Saini, Dr. K.K. Saini

1Deenbandhu Chhoturam University of Science and Technology Murthal, Sonepat, Haryana, India
2Hindu College of Engineering, Sonepat, Haryana, India

Abstract
Phonocardiograph is a continuous, non-invasive, low cost and accurate recording and monitoring of valve functioning of heart using heart sound signal. When Hilbert Transform is directly applied, an Analytic Signal of Heart Sound Signal is obtained. In order to decipher the details of the signal, Hilbert-Huang Transform is applied on the Heart Sound Signal. The HHT has two step analyses: The first one is Empirical Mode Decomposition (EMD) and the second part is Hilbert Spectral Analysis (HSA). The HHT analysis has been applied for pre-exercise and post-exercise data acquired from a subject. The subject had to go for stair-exercise (75 stepped stairs) for this purpose.

Keyword
Hilbert Transform, HHT, Intrinsic Mode Functions, Empirical Mode Decomposition, Hilbert Spectrum

I. Introduction
Signal processing is most growing and a area having emerging technology now a days. Signal processors are sometimes used for radio, transportation, medicine and production field etc. In 1743 a renowned Swiss mathematician Leonard Euler (1707-1783) derived the formula:

\[ e^{iz} = \cos(z) + j \sin(z) \]  (1)

After 150 years, physicist Arthur E. Kennelly and the scientist Charles P. Steinmetz used this formula to derive the complex notation for harmonic wave forms in electrical engineering as follows,

\[ e^{j\omega t} = \cos(\omega t) + j \sin(\omega t) \]  (2)

Later on, in starting of 20th century, the German scientist David Hilbert (1862-1943) proved that the function \( \sin(\omega t) \) is simply Hilbert transform of \( \cos(\omega t) \). This produced the \( \pm \pi/2 \) phase-shift operator which is an elementary property of the Hilbert transform.

A. Hilbert Transform
In time domain, The Hilbert transform of any function \( y(t) \) is defined for all \( t \) by,

\[ \hat{y}(t) = \frac{1}{\pi} \lim_{\epsilon \to 0} \int_{-\infty}^{+\infty} \frac{y(t)}{t-t'} dt \]  (3)

Where, the \( \epsilon \) in front of the integral denotes the Cauchy principal value which expands the class of functions for which the integral exist [1]. In other words Hilbert transform of an incoming signal may be defined as a convolution between the signal and the Hilbert transformer \( h(t) \), i.e.

\[ \hat{y}(t) = h(t) * y(t) \]  (4)

Where \( * \) represents convolution;

\( \hat{y}(t) \) = The Hilbert Transform of \( y(t) \);
\( h(t) \) = The time impulse response of Hilbert transformer \( [ h(t)=1/ (\pi t) ]; \)
\( y(t) \) = A real continuous time domain input.
When talking of the frequency domain Hilbert transform may be defined as phase shift of \( \pm \pi/2 \) over the frequency range of the signal.

B. Hilbert-Huang Transform: An Ideal Tool for PCG analysis:
The development of the Hilbert-Huang Transform (HHT) was inspired by the need to describe non-linear, complex and distorted type of waveforms[4]. Following points can prove that HHT is ideal tool for Heart Sound (PCG) analysis:

• The Traditional data-analysis methods are based on stationary and linear assumptions.
• Since, in most real systems, either natural or even man-made ones, the data are found to be both nonlinear and non-stationary, mostly.
• A necessary condition to represent nonlinear and non-stationary data is that it should be adaptive.
• This method is adaptive which means the definition of the basis is data-dependent i.e. a posterior-defined basis.
• The HHT method is potentially much suitable for non-stationary, non-linear and especially for time-frequency-energy representation of data.
Since, Heart Sound (PCG) is also anon-stationary and non-linear signal. Hence, HHT is ideal tool for heart sound (PCG) analysis.

C. Huang-Hilbert Transforms (Modified Hilbert Transform):
This new method (HHT) has been developed for the analysis of nonlinear and non-stationary data. The important part of this method is “Empirical mode decomposition” i.e. EMD. Using EMD, any complex and complicated data could be decomposed into a finite and small number of “intrinsic mode functions” i.e., IMFs. These IMFs admit well-behaved Hilbert transforms. Such kind of decomposition method is much adaptive and highly efficient. As this decomposition method is completely based on the some local characteristic of time scale of the data, therefore, it’s applicable to nonlinear as well as non-stationary processes. The implementation of the Hilbert transform on obtained IMFs, yield instantaneous frequencies as functions of time. These instantaneous frequencies are having sharp identifications of imbedded information. The final result is presented in the form of energy-frequency-time distribution, which is also called as Hilbert Spectrum. In this complete method, the central theme is the introduction of the “intrinsic mode functions” that is based on local characteristics of the signal, which makes the instantaneous Frequency much informative[5],[19],[20].
**D. Hilbert Spectrum:**

After obtaining the components of intrinsic mode function, it would be easier to apply the Hilbert transform to each IMFs followed by computation of the instantaneous frequency. By performing the Hilbert transform on each IMF, the data can be expressed in following form:

\[ x(t) = \sum_{n=0}^{n} a_n(t) \cdot e^{i \omega(t) dt} \]  

(5)

Above equation is also useful for representation of amplitude and instantaneous frequency as functions of time in a 3-D plot, in which the amplitude can be contoured on the plane of frequency-time. This frequency-time distribution of the amplitude is called as the Hilbert amplitude spectrum, \( \tilde{H}(\omega, t) \), i.e. “Hilbert spectrum”.

**II. Implementation**

**A. Implementation of Hilbert Transform on PCG signal:**

The platform used to implement Hilbert transform to the Heart Sound (PCG) signal is MATLAB 7.4.

As far as Hilbert Transform is concerned there is an in built function of finding Hilbert in MATLAB. Such that if \( \text{x} \) is the array which stores the samples of the Heart sound (PCG) signal. And \( \text{y} \) is an array that stores complex samples where the real part is given by the corresponding sample in the array \( \text{x} \) and the imaginary value is that of the Hilbert transform of signal. In other words \( \text{y} \) is the analytic signal formed by the Hilbert transform of the signal \( \text{x} \).

Here, the Fig.1 shows the original Heart Sound Signal, which is taken for analysis. The Fig.2 shows the Analytic signal of original Heart Sound signal. The analytic representation is a generalization of the phasor form of waveform. After making Heart Sound signal Analytic, we can look it as a rotating vector having instantaneous phase \( \Phi(t) \) and instantaneous amplitude \( A(t) \) in the time domain. Since the any signal and its Hilbert transform are always mutually orthogonal (i.e. phase shift occurs by \( \pi/2 \)), hence, the analytic seems to be in “Spiral shape” (as in Fig.2). Hence, the finally obtained Analytic signal is superimposition of original signal and its Hilbert Transform with respect to time axis which is clearly visible by its three dimensional plot.

**B. Implementation of Hilbert-Huang Transform on Heart Sound Signal**

Since Heart Sound (PCG) signal is a non-stationary & non-linear signal so directly applying Hilbert transform to the Heart Sound signal to find instantaneous frequency is of low significance. Hence we need to decompose the Heart Sound signal into simpler Intrinsic Mode Functions (IMFs) so that we can minutely identify and analyze the frequency contents of the signal. As discussed earlier that the Hilbert-Huang Transform analysis consists of two section:

- Empirical Mode Decomposition (EMD) of Signal, in which we decompose the original Heart Sound Signal into IMF's.
- Hilbert Spectral Analysis (HSA), in which we apply the Hilbert transform on the different components (IMFs), obtained after decomposition of original HS signal and find out the Hilbert Spectrum (i.e. Energy-Frequency-Time plot) finally.

**C. Empirical Mode Decomposition of Heart Sound (PCG) Signal:**

As shown in Fig. 3, the Empirical Mode Decomposition is applied on original Heart Sound Signal to obtain Intrinsic Mode Functions. Total 12 IMF's are obtained after decomposition. The twelfth IMF is not an IMF; it’s a residue which is monotonic function.
D. Applications of Hilbert Transform to IMFs after EMD:
Since we have decomposed the Original Heart Sound signal and got the IMFs. Now after applying the Hilbert Transform on IMFs obtained, we can easily analyse that which frequencies occurred and when during the signal duration, and also shows at a glance that where most of the signal energy is concentrated in time and frequency space. Hence in this particular case (which is shown in Fig. 3) we can get 12 analytic signals after applying the Hilbert Transform on each IMF, from which we can find the instantaneous frequency profile of each IMF.

E. Construction of Hilbert Spectrum:
As it is described earlier that by construction of Hilbert Spectrum one can easily analyse that which frequencies occurred and when, during the signal duration, and it also shows at a glance that where most of the signal energy is concentrated in the time and the frequency space. Hence, in Hilbert Spectrum, a 3-D plot may be constructed of the amplitude at a given time and frequency.

III. Results and Conclusions
Now, the Hilbert-Huang Transform is applied to the Heart Sound data. Through this analysis we intend to find the effects of Postural and situation changes on the Heart Sound data of the subject. Therefore first we consider a healthy subject and record the Heart Sound (PCG) while the subject is in lying position and relaxed stage. Then, we record the subject’s data (i.e. pre-exercise data) and apply the Hilbert-Huang Transform to find the Energy-Frequency-Time plot (Hilbert Spectrum). After this we make the subject to have stair-exercise (to go upstairs and downstairs through 75 steps stairs). Then, we repeat the above whole procedure to find the Energy-Frequency-Time plot for the subject (i.e. post-exercise data analysis). Hence, the obtained two plots are shown in Fig. 5 Looking at Hilbert Spectrum in Fig. 5 (a), it is evident that, the dominant-energy-frequency band of signal is constituted by the higher frequency (HF) components. Now, observing the the energy spectrum if Fig. 5 (b) we find noticeable change in the instantaneous frequency of the IMFs. Here, we can conclude that, the dominant-energy-frequency band of signal is constituted by lower frequency (LF). This occurs because; respiration signal and a little bit muscles noise create interference with Heart sound signal and makes the dominant-energy-frequency components into lower frequency (LF) region.

Now, a subject of age group 76-87 years has been considered and heart sound has been recorded while subject in the laying position and relaxed stage. Then Huang-Hilbert Transform is applied to find the Hilbert Spectrum of the Heart Sound data of the aged subject. Now, we compare the Hilbert spectrum of this aged subject with that of another healthy subject of age group 19-28 years. The both plots are shown in Fig. 6.
Looking at the Hilbert spectrum in Fig. 6(a), it is evident that, the dominant-energy-frequency range of signal is constituted by the low frequency (LF) components. And when we observe at the Fig. 6(b), we find that, the dominant-energy-frequency range constituted by the higher frequency (HF) components. This information suggests the better situation of heart valves of normal subject of age group (19-28 years) in comparison to that of aged (age group 76-87 years) subject. This frequency variation also suggests some problem in working of circulatory system and Heart Valves of aged subject. The original heart sound and ECG signal of normal subject (age group, 19-28 years) and aged subject (age group, 76-87 years) have been shown in Fig. 7.

IV. Future Scope
The Hilbert-Huang Transform analysis could be implemented to study minutely the impact on heart sound & its Hilbert Spectrum due to various postural and subject’s situation changes. There is also a future scope to study and distinguish the spectral changes of Heart Sound due to different types of Heart Murmurs e.g. aortic valve stenosis, Aortic Regurgitation, Atrial Septal Defect (ASD), Mitral Regurgitation and Ventricular Septal Defect (VSD) etc.

References
[9] Lehner R. J., R. M.Rangayyan,“A three-channel microcomputer system for segmentation and characterization of


