PAPR Reduction in OFDM Systems with Various Point FFTs

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Abstract
Orthogonal Frequency Division Multiplexing (OFDM) is considered to be a promising technique against the multipath fading channel for wireless communications. However, OFDM faces the Peak-to-Average Power Ratio (PAPR) problem that is a major drawback of multicarrier transmission system which leads to power inefficiency in RF section of the transmitter. This paper presents different PAPR reduction techniques and concludes an overall comparison of these techniques. Simulated results are also included for the modulation technique (BPSK, QPSK) with different point FFTs.

Keywords
OFDM, PAPR, CDF, BPSK, QPSK

I. Introduction
The basic principle of OFDM is to split a high-rate data stream into a number of lower rate streams that are transmitted simultaneously over a number of subcarriers. These subcarriers are overlapped with each other. Because the symbol duration increases for lower rate parallel subcarriers, the relative amount of dispersion in time caused by multipath delay spread is decreased.

A. Peak–To–Average Power Ratio
Presence of large number of independently modulated sub-carriers in an OFDM system the peak value of the system can be very high as compared to the average of the whole system. This ratio of the peak to average power value is termed as Peak to Average Power Ratio. The formula for PAPR is as follows

$$PAPR = \frac{max|x(t)|^2}{E[|x(t)|^2]}$$

Coherent addition of N signals of same phase produces a peak which is N times the average signal. The major disadvantages of a high PAPR are:

- Increased complexity in the analog to digital and digital to analog converter.
- Reduction is efficiency of RF amplifiers.

A non-constant envelope with high peaks is a main disadvantage of Orthogonal Frequency Division Multiplexing (OFDM). These high peaks produce signal excursions into non-linear region of operation of the Power Amplifier (PA) at the transmitter, thereby leading to non-linear distortions and spectral spreading. Many Peak to Average Power Ratio (PAPR) reductions methods have been proposed in the literature. The objective of this review is to give a clear understanding of different techniques to reduce PAPR of the signal.

1. Baseband PAPR
Continuous-time PAPR: In general, the PAPR of OFDM signals s(t) is defined as the ratio between the maximum instantaneous power and its average power

$$PAPR[x(t)] = [\, 0 \le t \le NT[x(t)^2\,]] \,/\, Pav$$

2. Passband PAPR
OFDM system usually does not employ pulse shaping, since the power spectral density of the band-limited OFDM signal is approximately rectangular. Thus, the amplitude of OFDM RF signals can be expressed as

$$xPB(t) = \{\Re\{x(t)e^{j2\pi fct}\}$$

It effectively combats the multipath fading channel and improves the bandwidth efficiency. At the same time, it also increases system capacity so as to provide a reliable transmission.

B. Cumulative Distribution Function
The Cumulative Distribution Function (CDF) is one of the most regularly used parameters, which is used to measure the efficiency of any PAPR technique. Normally, the Complementary CDF
(CCDF) is used instead of CDF, which helps to measure the probability that the PAPR of a certain data block exceeds the given threshold. By implementing the Central Limit Theorem for a multicarrier signal with a large number of subcarriers, the real and imaginary part of the time domain signals have a mean of zero and a variance of 0.5, which follows a Gaussian distribution. So Rayleigh distribution is followed for the amplitude of the multi-carrier signal, where as a central chi-square distribution with two degrees of freedom is followed for the power distribution of the system The Cumulative Distribution Function (CDF) of the amplitude z of an OFDM signal sample [11] is given by:

$$F(z) = 1 - \exp(z)$$

The CDF of the PAPR for an OFDM data block can be found as
$CDF = 1 - P(PAPR < z) = (1 - e^{-z})^{\alpha N}$
Where, N is the number of sub-carriers. For an oversampled OFDM, this formula must be advanced to
$CDF = (1 - e^{-z})^{\alpha N}$
Where, the PAPR of an oversampled signal for N subcarriers is approximated by the distribution for αN subcarriers without oversampling. For four times oversampled OFDM signals, α=2.3 is a good approximation.
Therefore, CCDF of the PAPR for an oversampled OFDM data block is:
$CCDF = 1 - CDF = 1 - (1 - e^{-z})^{\alpha N}$
The CCDF curve shows that how much time a signal spent at or above a given power level. The power level is expressed in dB relative to the average power.
The percentage of time the signal spends at or above each line defines the probability for that particular power level.

$$F(z) = 1 - \exp(z)$$

$$\begin{aligned} P(PAPR > z) &= 1 - P(PAPR < z) \\ &= 1 - F(z)^N \\ &= 1 - (1 - \exp(-z))^N \end{aligned}$$

OFDM consists of lots of independent modulated subcarriers, as a result the amplitude of such a signal can have very large values. These large peaks increase the amount of intermodulation distortion resulting in an increase in the error rate. The PAPR of an OFDM signal can be reduced in several ways: selective mapping, Golay sequences, cyclic coding, clipping and filtering, and multiple signal representation techniques.

II. Proposed Methodology

It should be noted that each different type of modulation has its own value for the error function. This is because each type of modulation performs differently in the presence of noise. In particular, higher order modulation schemes (e.g. 64QAM, etc) that are able to carry higher data rates are not as robust in the presence of noise. Lower order modulation formats (e.g. BPSK, QPSK, etc.) offer lower data rates but are more robust. Here we used MATLAB for simulating a random BPSK/QPSK modulation with and without the j rotation for various point FFTs. The script performs the following as per the flow chart shown in Fig. 1:

1. Select Input Size of FFT, No. of Sub-carriers and No. of Symbols etc.
2. Generate BPSK & QPSK Signals.
3. Assign modulated symbols to Subcarriers of both methods.
4. Take Inverse FFT.
5. Compute Peak to Average Power Ratio for all Four cases (BPSK & QPSK with & without rotation of j)
6. Plot the PAPR graph between CDF & PAPR values for all cases.

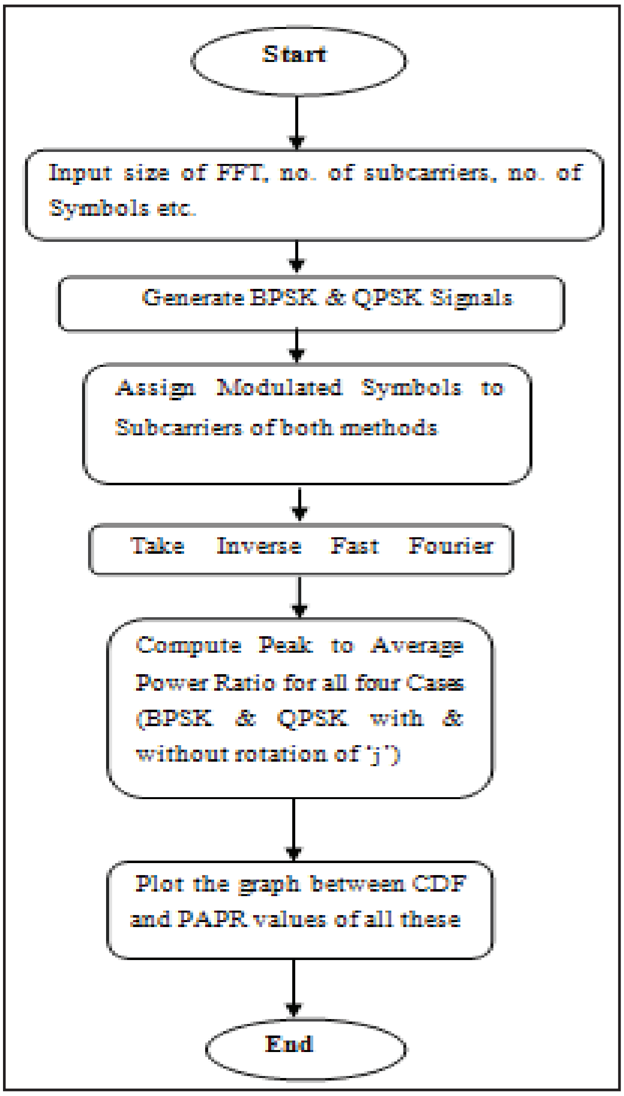

Fig. 1: Flow Chart of Proposed Methodology

III. Result

In the result we have shown the graphs showing PAPR for 128 Point, 256 point & 512 point FFTs. Also the effect of no. of subcarriers is shown in the graph & its comparison is shown in the given Table 4 & 8 respectively.

Table 4 shows the comparison of PAPR of various methods for different point of FFT & we have seen that performance of 128 point FFT is best as compared to 256 point & 512 point FFT, which is also shown in the graph. Table 8 shows the performance of PAPR of various methods for 128 point FFT for various subcarriers and we have observed that for 300 sub carriers we get the best results as compared to 100 & 200 subcarriers, which are also shown in the graph. It has been observed that as soon as we increased no. of subcarriers the performance of systems improves & PAPR reduces gradually.

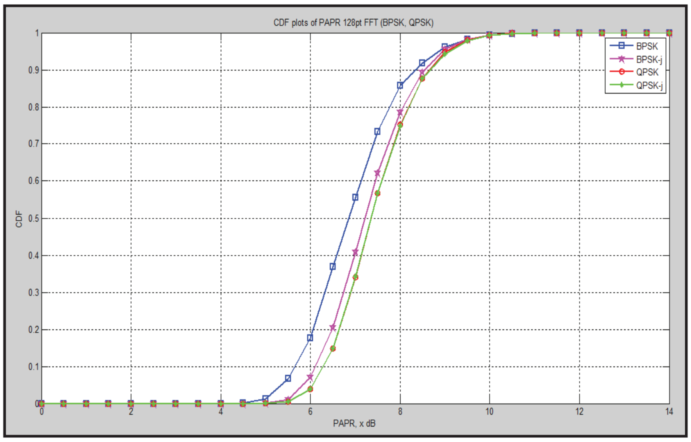

Fig. 1: PAPR for 128 point FFT with 100 sub-carriers

Fig. 1 shows the graph which is having PAPR values on X axis and CDF values at Y axis for various modulation techniques shown by different colors.

Table 1: PAPR for 128 point FFT with 100 sub-carriers

Technique → / CDF ↓	BPSK	BPSK-j	QPSK	QPSK-j
0.1	5.65	6.12	6.28	6.28
0.2	6.06	6.48	6.63	6.63
0.3	6.32	6.73	6.9	6.9
0.4	6.59	6.97	7.13	7.13
0.5	6.85	7.22	7.35	7.35
0.6	7.13	7.45	7.6	7.6
0.7	7.41	7.75	7.87	7.87
0.8	7.76	8.07	8.2	8.2
0.9	8.35	8.57	8.67	8.67

Table 1 shows the results of PAPR values at different CDF points of various modulation techniques in 128 point FFT with 100 subcarriers as shown in fig. 1.

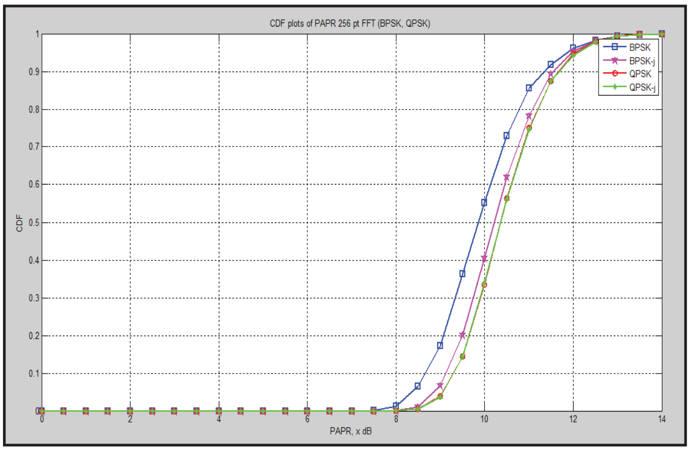

Fig. 2: PAPR for 256 point FFT with 100 sub-carriers

Fig 2 shows the graph which is having PAPR values on X axis and CDF values at Y axis for various modulation techniques shown by different colors.

Table 2: PAPR for 256 point FFT with 100 sub-carriers

Technique →	BPSK	BPSK-j	QPSK	QPSK-j
CDF ↓				
0.1	8.67	9.13	9.3	9.3
0.2	9.08	9.5	9.65	9.65
0.3	9.34	9.74	9.92	9.92
0.4	9.6	10.0	10.1	10.1
0.5	9.86	10.2	10.4	10.4
0.6	10.1	10.5	10.6	10.6
0.7	10.4	10.8	10.9	10.9
0.8	10.8	11.1	11.2	11.2
0.9	11.4	11.6	11.7	11.7

Table 2 shows the results of PAPR values at different CDF points of various modulation techniques in 256 point FFT with 100 subcarriers as shown in Fig. 2.

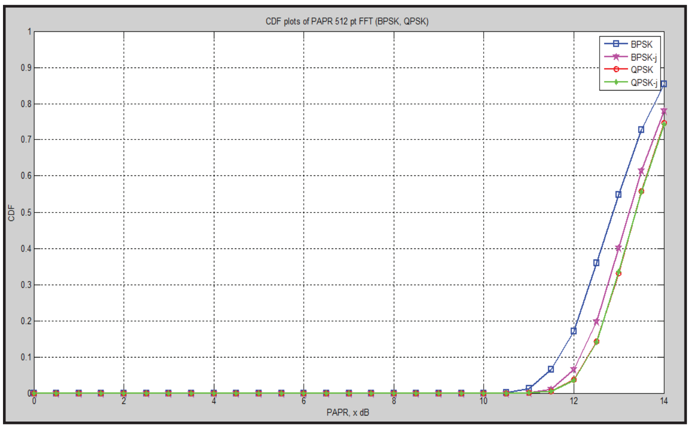

Fig. 3: PAPR for 512 point FFT with 100 sub-carriers

Fig. 3 shows the graph which is having PAPR values on X axis and CDF values at Y axis for various modulation techniques shown by different colors.

Table 3: PAPR for 512 point FFT with 100 sub-carriers

Technique →	BPSK	BPSK-j	QPSK	QPSK-j
CDF ↓				
0.1	11.7	12.2	12.3	12.3
0.2	12.1	12.5	12.7	12.7
0.3	12.3	12.8	12.9	12.9
0.4	12.6	13.0	13.1	13.1
0.5	12.9	13.2	13.4	13.4
0.6	13.1	13.5	13.6	13.6
0.7	13.4	13.8	13.9	13.9
0.8	13.8	-	-	-

Table 3 shows the results of PAPR values at different CDF points of various modulation techniques in 512 point FFT with 100 subcarriers as shown in fig. 3.

Table 4: Comparison of PAPR among given point of FFT

Name of FFT →	128 point FFT	256 point FFT	512 point FFT
Technique↓			
BPSK	5.65	8.67	11.7
BPSK-j	6.12	9.13	12.2
QPSK	6.28	9.3	12.3
QPSK-j	6.28	9.3	12.3

Table 4: Shows the comparison among 128,256 and 512 point FFTs. It shows the value of PAPR at 0.1 CDF because at 0.1 CDF the value of PAPR is changes in all PAPR reduction techniques so it becomes a standard value to check PAPR also we want the most reduced value of PAPR. After analyzing the given table it is concluded that best result is obtained for 128 point FFT.

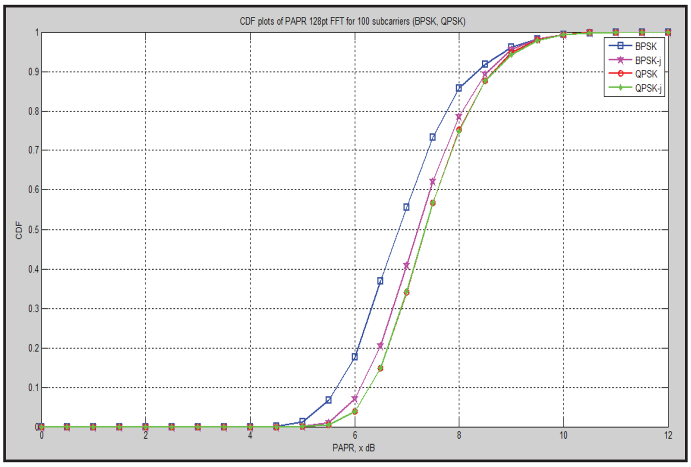

Fig. 4 PAPR for 128 point FFT with 100 sub-carriers

Fig. 4 shows the graph which is having PAPR values on X axis and CDF values at Y axis for various modulation techniques shown by different colors.

 ISSN : 2230-7109 (Online) | ISSN : 2230-9543 (Print)

Table 5: PAPR for 128 point FFT with 100 sub-carriers

Technique → / CDF↓	BPSK	BPSK-j	QPSK	QPSK-j
1	5.65	6.12	6.28	6.28
0.2	6.06	6.48	6.63	6.63
0.3	6.32	6.73	6.9	6.9
0.4	6.59	6.97	7.13	7.13
0.5	6.85	7.22	7.35	7.35
0.6	7.13	7.45	7.6	7.6
0.7	7.41	7.75	7.87	7.87
0.8	7.76	8.07	8.2	8.2
0.9	8.35	8.57	8.67	8.67

Table 5: shows the results of PAPR values at different CDF points of various modulation techniques in 128 point FFT with 100 subcarriers as shown in Fig .4.

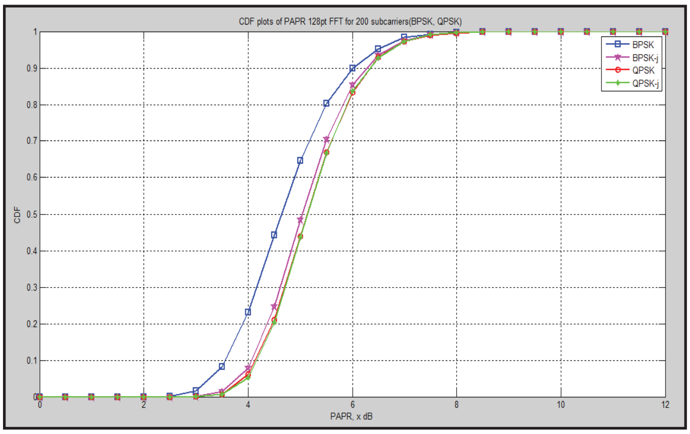

Fig. 5 PAPR for 128 point FFT with 200 sub-carriers

Fig. 5 shows the graph which is having PAPR values on X axis and CDF values at Y axis for various modulation techniques shown by different colors.

Table 6: PAPR for 128 point FFT with 200 sub-carriers

Technique → / CDF ↓	BPSK	BPSK-j	QPSK	QPSK-j
0.1	3.56	4.07	4.13	4.16
0.2	3.9	4.37	4.47	4.5
0.3	4.16	4.62	4.71	4.71
0.4	4.4	4.83	4.92	4.92
0.5	4.65	5.05	5.14	5.14
0.6	4.9	5.28	5.36	5.36
0.7	5.19	5.5	5.61	5.61
0.8	5.49	5.82	5.91	5.89
0.9	6.0	6.28	6.34	6.34

Table 6 shows the results of PAPR values at different CDF points of various modulation techniques in 128 point FFT with 200 subcarriers as shown in fig. 5.

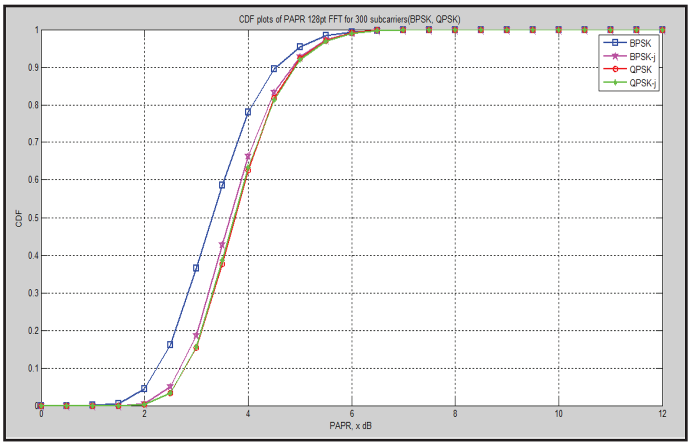

Fig. 6: PAPR for 128 point FFT with 300 sub-carriers

Fig. 6 shows the graph which is having PAPR values on X axis and CDF values at Y axis for various modulation techniques shown by different colors.

Table 7: PAPR for 128 point FFT with 300 sub-carriers

Technique → / CDF ↓	BPSK	BPSK-j	QPSK	QPSK-j
0.1	2.31	2.68	2.78	2.8
0.2	2.57	3.02	3.1	3.11
0.3	2.81	3.24	3.33	3.34
0.4	3.05	3.46	3.55	3.55
0.5	3.27	3.67	3.74	3.74
0.6	3.5	3.88	3.94	3.94
0.7	3.79	4.14	4.2	4.2
0.8	4.09	4.42	4.48	4.46
0.9	4.53	4.86	4.9	4.9

Table 7 shows the results of PAPR values at different CDF points of various modulation techniques in 128 point FFT with 300 subcarriers as shown in fig. 6.

Table 8: Performance of PAPR for 128 point FFT for various Sub-carriers

	100 sub carriers	200 sub carriers	300 sub carriers
BPSK	5.65	3.56	2.31
BPSK with 'j'	6.12	4.07	2.68
QPSK	6.28	4.13	2.78
QPSK with 'j'	6.28	4.16	2.8

Table 8: Shows the performance of 128 point FFT for different numbers of Sub-carriers. It shows the value of PAPR at 0.1 CDF because at 0.1 CDF the value of PAPR is changes in all PAPR reduction techniques so it becomes a standard value to check PAPR and also we want most reduced PAPR value. After analyzing the given table it is concluded that the best result is obtained for 300 subcarriers.

IV. Conclusion

This paper summarizes the overall comparison of modulation techniques of PAPR reduction. Simulation of modulation technique like BPSK & QPSK with & without j rotation is executed here for various point FFTs. It is analyzed after simulation that 128 point FFT produces best result for reduced PAPR among 128,256 & 512 point FFTs. And also 128 point gave best result for 300 subcarriers. It means we get a good result with large number of subcarriers. The best outcome of 128 point FFT with 300 subcarriers is 2.31 for BPSK technique without j rotation. We have explored a probable way of reducing PAPR (peak to average power ratio) in OFDM by changing the phase of some of the subcarriers.

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