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High-Order Finite Difference Modeling and Simulation Analysis of Microstrip Patch Antenna

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Abstract

In this paper, higher order finite difference scheme namely FDTD (2,4) scheme has been used for analysis of Microstrip Patch Antenna. The Uniaxial Perfectly Matched Layer (UPML) has been used as an absorbing boundary condition to terminate the computational domain. To justify higher order computational efficiency achieved by this method, the results have been compared with the conventional FDTD (2,2) method.

Keywords

FDTD (2,2), FDTD(2,4), UPML.

I. FDTD(2,4) scheme

The FDTD (2,4) scheme uses central finite differences which is fourth-order-accurate in space and second-order-accurate in time [2]. Using taylor series expansion, we have

$$F(x+\Delta x,y,z)=F(x,y,z)+\Delta x F'(x,y,z)+\frac{\Delta x^2}{2!}F''(x,y,z)+\frac{\Delta x^3}{3!}F'''(x,y,z)+\ldots \quad (1)$$

$$F(x+\frac{\Delta x}{2},y,z)=F(x,y,z)+\frac{\Delta x}{2}F'(x,y,z)+\frac{\Delta x^2}{8}F''(x,y,z)+\frac{\Delta x^3}{48}F'''(x,y,z)+\ldots \quad (2)$$

$$F(x-\frac{\Delta x}{2},y,z)=F(x,y,z)-\frac{\Delta x}{2}F'(x,y,z)+\frac{\Delta x^2}{8}F''(x,y,z)-\frac{\Delta x^3}{48}F'''(x,y,z)+\ldots \quad (3)$$

$$F(x+3\frac{\Delta x}{2},y,z)=F(x,y,z)+3\frac{\Delta x}{2}F'(x,y,z)+9\frac{\Delta x^2}{8}F''(x,y,z)+27\frac{\Delta x^3}{48}F'''(x,y,z)+\ldots \quad (4)$$

$$F(x-3\frac{\Delta x}{2},y,z)=F(x,y,z)-3\frac{\Delta x}{2}F'(x,y,z)+9\frac{\Delta x^2}{8}F''(x,y,z)-27\frac{\Delta x^3}{48}F'''(x,y,z)+\ldots \quad (5)$$

using above equations, the first derivative of F(x, y, z) wth respect to x can be found as:

$$\frac{\partial F(x,y,z)}{\partial x}\approx\frac{9}{8}\frac{(F(x+\frac{\Delta x}{2},y,z)-F(x-\frac{\Delta x}{2},y,z))}{\Delta x}-\frac{1}{24}\frac{(F(i+\frac{3\Delta x}{2},y,z)-F(x-\frac{3\Delta x}{2},j,k))}{\Delta x}+O(\Delta x^4) \quad (6)$$

More accuracy in approximation to derivatives are given by higher-order finite-difference schemes. The derivation of Courant stability criterion for the FDTD (2,4) scheme was given by Fang [3] as

$$\Delta t\le\frac{6}{7}\frac{1}{c\sqrt{\frac{1}{\Delta x^2}+\frac{1}{\Delta y^2}+\frac{1}{\Delta z^2}}} \quad (7)$$

This gives limit on maximum time step size that can be used for simulation. Numerical experiments shows that higher order FDTD suffers comparatively less numerical dispersion as compared to standard FDTD (2,2) method. In next section, broadband analysis of microstrip patch antenna is carried out using 3D FDTD(2,4)-UPML method and results have been compared with conventional FDTD and expected theoretical result.

II. 3D-FDTD UPML Formulation

The Uniaxial Perfectly Matched Layer (UPML) is artificial anisotropic absorbing material that is theoretically designed to allow no reflections regardless of the frequency, any polarization and angle of incidence of any plane wave upon it's interface. Thus, it is type of Absorbing Boundary Condition (ABC) that is used to terminate the computational domain. The maxwell's equation in an anisotropic medium can be expressed as follows:

$$\nabla\times\vec{H}=j\omega\varepsilon\bar{\bar{s}}\vec{E} \quad (8)$$

$$\nabla\times\vec{E}=-j\omega\bar{\bar{s}}\mu\vec{H} \quad (9)$$

Here, $\vec{E}$ and $\vec{H}$ are the electric and magnetic field intensity vectors repectively with components in phasor form and $\bar{\bar{s}}$ is diagonal tensor given as

$$\bar{\bar{s}}=\begin{bmatrix}\frac{s_y s_z}{s_x} & 0 & 0\\ 0 & \frac{s_x s_z}{s_y} & 0\\ 0 & 0 & \frac{s_x s_y}{s_z}\end{bmatrix}$$

Here, s_x, s_y and s_z represents relative complex permittivities along x, y and z directions. They are given as follows:

$$s_x=k_x+\frac{\sigma_x}{j\omega\varepsilon} \quad (10)$$

$$s_y=k_y+\frac{\sigma_y}{j\omega\varepsilon} \quad (11)$$

$$s_z=k_z+\frac{\sigma_z}{j\omega\varepsilon} \quad (12)$$

The equation (8) and (9) can be given in three scalar PDEs as

$$\begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = j\omega\varepsilon \begin{bmatrix} \frac{s_y s_z}{s_x} & 0 & 0 \\ 0 & \frac{s_x s_z}{s_y} & 0 \\ 0 & 0 & \frac{s_x s_y}{s_z} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} \quad (13)$$

The relation between electric flux density and electric field intensity components are given as:

$$\vec{D}_x = \varepsilon \frac{s_z}{s_x} \vec{E}_x \quad (14)$$

$$\vec{D}_y = \varepsilon \frac{s_x}{s_y} \vec{E}_y \quad (15)$$

$$\vec{D}_z = \varepsilon \frac{s_y}{s_z} \vec{E}_z \quad (16)$$

Substituting (14), (15) and (16) into (13) results in

$$\begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = j\omega \begin{bmatrix} s_x & 0 & 0 \\ 0 & s_y & 0 \\ 0 & 0 & s_z \end{bmatrix} \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} \quad (17)$$

The above mentioned frequency domain PDEs can be converted into time domain PDEs using transformation $j\omega \rightarrow \frac{\partial}{\partial t}$.

$$\begin{bmatrix} \frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z} \\ \frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x} \\ \frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y} \end{bmatrix} = \frac{\partial}{\partial t} \begin{bmatrix} k_y & 0 & 0 \\ 0 & k_z & 0 \\ 0 & 0 & k_x \end{bmatrix} \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} +$$

$$\frac{1}{\varepsilon} \begin{bmatrix} \sigma_y & 0 & 0 \\ 0 & \sigma_z & 0 \\ 0 & 0 & \sigma_x \end{bmatrix} \begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} \quad (18)$$

Using same analogy, equation (9) can be reformulated as

$$\begin{bmatrix} \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \\ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \\ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \end{bmatrix} = -\frac{\partial}{\partial t} \begin{bmatrix} k_y & 0 & 0 \\ 0 & k_z & 0 \\ 0 & 0 & k_x \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} -$$

$$\frac{1}{\mu} \begin{bmatrix} \sigma_y & 0 & 0 \\ 0 & \sigma_z & 0 \\ 0 & 0 & \sigma_x \end{bmatrix} \begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} \quad (19)$$

The time domain PDEs for relating electric field intensity and electric flux density can be rearranged and calculated as

$$\frac{\partial}{\partial t}(k_x D_x) + \frac{\sigma_x}{\varepsilon} D_x = \varepsilon[\frac{\partial}{\partial t}(k_z E_x) + \frac{\sigma_z}{\varepsilon} E_x] \quad (20)$$

$$\frac{\partial}{\partial t}(k_y D_y) + \frac{\sigma_y}{\varepsilon} D_y = \varepsilon[\frac{\partial}{\partial t}(k_x E_y) + \frac{\sigma_x}{\varepsilon} E_y] \quad (21)$$

$$\frac{\partial}{\partial t}(k_z D_z) + \frac{\sigma_z}{\varepsilon} D_z = \varepsilon[\frac{\partial}{\partial t}(k_y E_z) + \frac{\sigma_y}{\varepsilon} E_z] \quad (22)$$

Similarly, the time domain PDEs for relating magnetic field intensity and magnetic flux density can be rearranged and calculated as

$$\frac{\partial}{\partial t}(k_x B_x) + \frac{\sigma_x}{\varepsilon} B_x = \mu[\frac{\partial}{\partial t}(k_z H_x) + \frac{\sigma_z}{\varepsilon} H_x] \quad (23)$$

$$\frac{\partial}{\partial t}(k_y B_y) + \frac{\sigma_y}{\varepsilon} D_y = \mu[\frac{\partial}{\partial t}(k_x H_y) + \frac{\sigma_x}{\varepsilon} H_y] \quad (24)$$

$$\frac{\partial}{\partial t}(k_z B_z) + \frac{\sigma_z}{\varepsilon} B_z = \varepsilon[\frac{\partial}{\partial t}(k_y H_z) + \frac{\sigma_y}{\varepsilon} H_z] \quad (25)$$

Thus, there are are 12 update equations i.e for D_x, D_y, D_z, E_x, E_y, E_z, B_x, B_y, B_z, H_x, H_y and H_z. All the update equations were carefully derived and are given in appendix for reference.

A. Computational Space Parameters

In FDTD, different materials can be modelled by varying values of k_x, k_y, k_z, σ_x, σ_y and σ_z. Isotropic material can be modelled by using $k_x = k_y = k_z = 1$, $\sigma_x = \sigma_y = \sigma_z = \sigma_{mat}$, where σ_{mat} is the conductivity of the material.

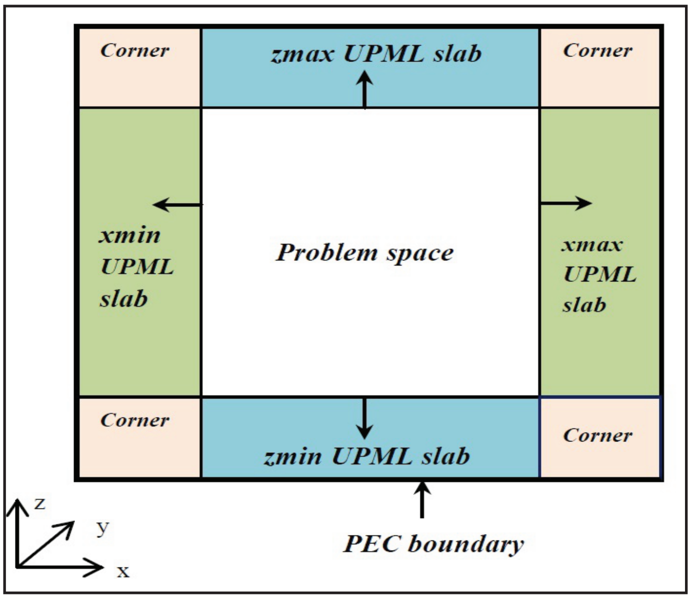

Fig. 1: Cross Sectional View of Computational Space

x_{min}, x_{max} UPML region: $\sigma_y = \sigma_z = 0$, $k_y = k_z = 1$. σ_x, k_x varies

y_{min}, y_{max} UPML region: $\sigma_x = \sigma_z = 0$, $k_x = k_z = 1$. σ_y, k_y varies

z_{min}, z_{max} UPML region: $\sigma_x = \sigma_y = 0$, $k_x = k_y = 1$. σ_z, k_z varies.

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For corner UPML region, a combination of two or more are used. For corners shown in fig. 1, σ_y, σ_z, k_y, k_z varies.
The parameter σ and k are graded geometrically along the normal axis of UPML region in order to obtain gradual attenuation of any incident wave on the UPML. The geometric grading equations are given as

$$\sigma_{x,y,z} = (g^{\frac{1}{\Delta}})^u \sigma_0 \qquad (26)$$

$$\kappa_{x,y,z}(u) = (g^{\frac{1}{\Delta}})^u \qquad (27)$$

Here, u is the normal distance between the point where the parameter is calculated and UPML computational space boundary. Δ is the spatial discretization interval, i.e Δ=max(Δx, Δy, Δz) and g, σ_0 are constant.

III. Broadband Analysis Using FDTD (2,4) Scheme

Three dimensional FDTD with UPML was implemented in MATLAB version 15b. The main objective was to simulate the rectangular patch antenna shown in Fig. 2. The length and width of the antenna are L=16.2mm W=12.45mm respectively. The height of the substrate and substrate dielectric constant are h = 0.795mm and 2.2 respectively [3]. The simulation results can be used to obtain S_{11} (dB) vs. frequency of this antenna.

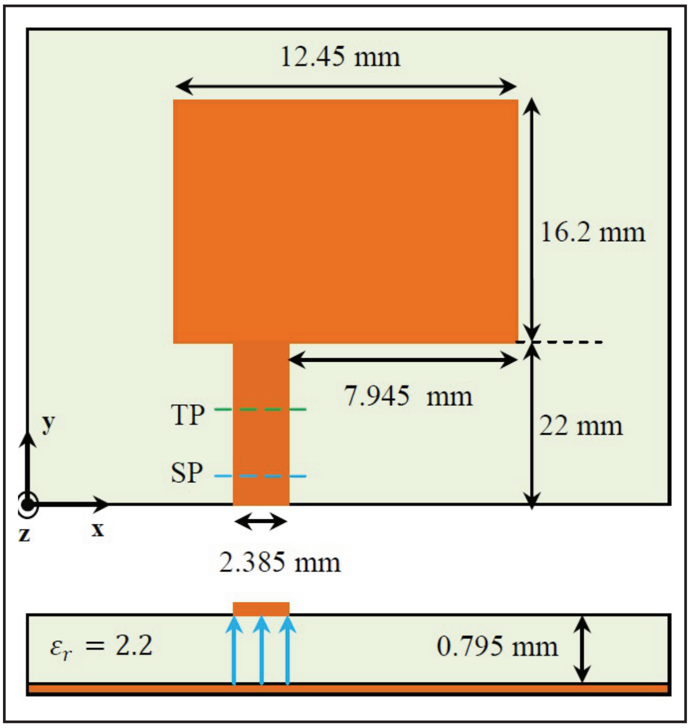

Fig. 2: Microstrip Line Fed Rectangular Patch Antenna (not to Actual Scale). Here, TP and SP Represents Terminal and Source Plane Respectively [5].

A. FDTD Simulation Details

The size of the problem space was 42× 101 ×5 cells. Since there were 10 layers of Uniaxial Perfectly matched Layers (UPML) adjoining each of the six faces of the problem space, the total size of the computational space was 60 × 119×23 cells. The cell size Δx, Δy and Δz were taken to be 0.389mm, 0.400mm and 0.265mm respectively. The time step size was used as per Courant stability criterion given by Fang [3].

B. Simulation Result

The response of the microstrip line fed patch antenna after 5488 steps of iterations has been shown in Fig. 3. It can be seen that there is significantly less distortion after the trailing edge of the Gaussian pulse, which in turn yield better accurate result. It can be noticed that once pulse has passed through the terminal plane, the fields are nearly zero. This is due to UPML, which will absorb the pulse. This shows the effectiveness of the UPML. Further, it is depicted from figure that the reflected wave can be obtained by simply subtracting the incident wave from total wave [4].

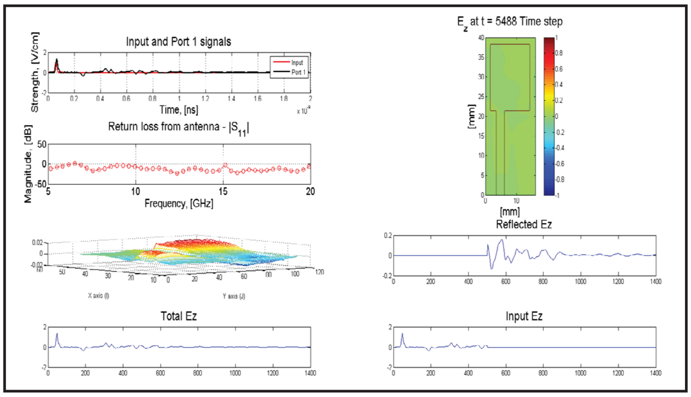

Fig. 3: Response of the Microstrip Line Fed Patch Antenna after 5488 steps of iterations [subtracting input Ez data upto 500 steps from the total Ez gives reflected wave]

C. Microstrip Antenna Resonant Frequency

Cavity model of microstrip patch antennas can be used to obtain approximated resonant frequencies values. The operating frequency of the antenna is obtained from the higher mode. The higher mode after the dominant mode is TM_{020}. The corresponding resonant frequency for this mode is given as

$$(f_r)_{020} = \frac{v_0}{2W\sqrt{\varepsilon_r}} \qquad (28)$$

where, v_0 is the velocity of light in free space. This value is calculated as $(f_r)_{020}$=8.1GHz. It can be seen from the Fig. 4 that the operating frequency of the cavity using high order FDTD(2,4) analysis is found to be 8.03GHz. This is in good agreement with actual value of operating frequency given by resonant cavity model as 8.1GHz [4]. Thus, there is a reasonable match between actual result and simulated result, especially the frequency at which resonance occurs. The table I demonstrates error of calculated operating frequency of cavity using Conventional FDTD, High Order FDTD (2,4) and theoretical value.

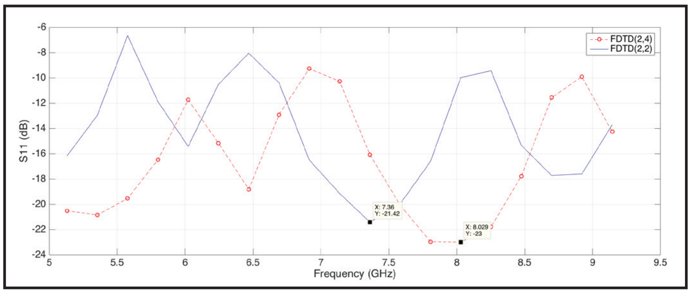

Fig. 4: Magnitude of S_{11} (dB) Versus Frequency for Microstrip Patch Antenna

Table 1: Comparison of operating frequency of cavity using conventional FDTD, Higher Order FDTD (2,4) method and theoretical calculation.

Theoretical value	FDTD(2,2)	Error	High Order FDTD (2,4) method	Error
8.1 GHz	7.36 GHz	9.13 %	8.03 GHz	0.86 %

IV. Conclusion

In this paper, a comparitive study has been made between FDTD(2,2) and FDTD(2,4) method. It has been found that the higher order FDTD (2,4) method yields more accurate result as compared to conventional FDTD (2,2) method. However, the simulation time required for FDTD (2,4) method is slightly more than FDTD (2,2). It is expected that there can be significant reduction in simulation if any unconditionally stable implicit FDTD method is used for the same analysis.

Appendix

[3D FDTD(2,4)-UPML update equation]

D_x update equation

$$D_x\Big|^{n+1}_{i+0.5,j,k} = \text{C1Dx}.\, D_x\Big|^{n}_{i,j+0.5,k} + \text{C2Dx}.(\frac{9}{8}\left(\frac{H_z\big|^{n+0.5}_{i+0.5,j+0.5,k}-H_z\big|^{n+0.5}_{i+0.5,j-0.5,k}}{\Delta y} - \frac{H_y\big|^{n+0.5}_{i+0.5,j,k+0.5}-H_y\big|^{n+0.5}_{i+0.5,j,k-0.5}}{\Delta z}\right) - \frac{1}{24}\left(\frac{H_z\big|^{n+0.5}_{i+0.5,j+1.5,k}-H_z\big|^{n+0.5}_{i+0.5,j-1.5,k}}{\Delta y} - \frac{H_y\big|^{n+0.5}_{i+0.5,j,k+1.5}-H_y\big|^{n+0.5}_{i+0.5,j,k-1.5}}{\Delta z}\right)$$

$$\text{C1Dx}=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right),\ \text{C2Dx}=\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right)$$

$$\text{C1Dx}=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right),\ \text{C2Dx}=\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right)$$

D_y update equation

$$D_y\Big|^{n+1}_{i,j+0.5,k} = \text{C1Dy}.\, D_y\Big|^{n}_{i,j+0.5,k} + \text{C2Dy}.(\frac{9}{8}\left(\frac{H_x\big|^{n+0.5}_{i,j+0.5,k+0.5}-H_x\big|^{n+0.5}_{i,j+0.5,k-0.5}}{\Delta z} - \frac{H_z\big|^{n+0.5}_{i+0.5,j+0.5,k}-H_y\big|^{n+0.5}_{i-0.5,j+0.5,k}}{\Delta x}\right) - \frac{1}{24}\left(\frac{H_x\big|^{n+0.5}_{i,j+0.5,k+1.5}-H_x\big|^{n+0.5}_{i,j+0.5,k-1.5}}{\Delta z} - \frac{H_z\big|^{n+0.5}_{i+1.5,j+0.5,k}-H_y\big|^{n+0.5}_{i-1.5,j+0.5,k}}{\Delta x}\right)$$

$$\text{C1Dy}=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right)\ \text{C2Dy}=\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right)$$

D_z update equation

$$D_z\Big|^{n+1}_{i,j,k+0.5} = \text{C1Dz}.\, D_y\Big|^{n}_{i,j,k+0.5} + \text{C2Dz}.(\frac{9}{8}\left(\frac{H_y\big|^{n+0.5}_{i+0.5,j,k+0.5}-H_y\big|^{n+0.5}_{i-0.5,j,k+0.5}}{\Delta x} - \frac{H_x\big|^{n+0.5}_{i,j+0.5,k+0.5}-H_x\big|^{n+0.5}_{i,j-0.5,k+0.5}}{\Delta y}\right) - \frac{1}{24}\left(\frac{H_y\big|^{n+0.5}_{i+1.5,j,k+0.5}-H_y\big|^{n+0.5}_{i-1.5,j,k+0.5}}{\Delta x} - \frac{H_x\big|^{n+0.5}_{i,j+1.5,k+0.5}-H_x\big|^{n+0.5}_{i,j-1.5,k+0.5}}{\Delta y}\right)$$

$$\text{C1Dz}=\left(\frac{2\varepsilon k_x-\sigma_x\Delta t}{2\varepsilon k_x+\sigma_x\Delta t}\right)\ \text{C2Dz}=\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_x+\sigma_x\Delta t}\right)$$

E_x update equation

$$E_z\Big|^{n+1}_{i+0.5,j,k} = \text{C1Ex}.\, E_x\Big|^{n}_{i+0.5,j,k} + \text{C2Ex}.\, D_x\Big|^{n+1}_{i+0.5,j,k} - \text{C3Ex}.\, D_x\Big|^{n}_{i+0.5,j,k}$$

$$\text{C1Ex}=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right)\ \text{C2Ex}=\left(\frac{2\varepsilon k_x+\sigma_x\Delta t}{\varepsilon(2\varepsilon k_z+\sigma_z\Delta t)}\right)$$

$$\text{C3Ex}=\left(\frac{2\varepsilon k_x-\sigma_x\Delta t}{\varepsilon(2\varepsilon k_z+\sigma_z\Delta t)}\right)$$

E_y update equation

$$E_y\Big|^{n+1}_{i,j+0.5,k} = \text{C1Ey}.\, D_y\Big|^{n}_{i,j+0.5,k} + \text{C2Ey}.\, D_y\Big|^{n+1}_{i,j+0.5,k} - \text{C3Ex}.\, D_y\Big|^{n}_{i,j+0.5,k}$$

$$\text{C1Ey}=\left(\frac{2\varepsilon k_x-\sigma_x\Delta t}{2\varepsilon k_x+\sigma_x\Delta t}\right)\ \text{C2Ey}=\left(\frac{2\varepsilon k_y+\sigma_y\Delta t}{\varepsilon(2\varepsilon k_x+\sigma_x\Delta t)}\right)$$

$$\text{C3Ey}=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{\varepsilon(2\varepsilon k_x+\sigma_x\Delta t)}\right)$$

E_z update equation

$$E_z\Big|^{n+1}_{i,j,k+0.5} = \text{C1Ez}.\, D_z\Big|^{n}_{i,j,k+0.5} + \text{C2Ez}.\, D_y\Big|^{n+1}_{i,j,k+0.5} - \text{C3Ez}.\, D_z\Big|^{n}_{i,j,k+0.5}$$

$$\text{C1Ez}=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right)\ \text{C2Ez}=\left(\frac{2\varepsilon k_z+\sigma_z\Delta t}{\varepsilon(2\varepsilon k_y+\sigma_y\Delta t)}\right)$$

$$\text{C3Ez}=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{\varepsilon(2\varepsilon k_y+\sigma_y\Delta t)}\right)$$

Bx update equation

$$B_x\Big|^{n+1.5}_{i,j+0.5,k+0.5} = \text{C1Bx}.\, B_x\Big|^{n+0.5}_{i,j+0.5,k+0.5} - \text{C2Bx}.(\frac{9}{8}\left(\frac{E_z\big|^{n+1}_{i,j+1,k+0.5}-E_z\big|^{n+1}_{i,j,k+0.5}}{\Delta y} - \frac{E_y\big|^{n+1}_{i,j+0.5,k+1}-E_y\big|^{n+1}_{i,j,k+0.5}}{\Delta z}\right) - \frac{1}{24}\left(\frac{E_z\big|^{n+1}_{i,j+1.5,k+0.5}-E_z\big|^{n+1}_{i,j+0.5,k+0.5}}{\Delta y} - \frac{E_y\big|^{n+1}_{i,j+0.5,k+1.5}-E_y\big|^{n+1}_{i,j,k+0.5}}{\Delta z}\right))$$

$$\text{C1Bx}=\left(\frac{2\varepsilon k_y-\sigma_y\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right)\ \text{C2Bx}=\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_y+\sigma_y\Delta t}\right)$$

By update equation

$$B_y\Big|^{n+1.5}_{i+0.5,j,k+0.5} = \text{C1By}.\, B_y\Big|^{n+0.5}_{i+0.5,j,k+0.5} - \text{C2By}.(\frac{9}{8}\left(\frac{E_x\big|^{n+1}_{i+0.5,j,k+1}-E_x\big|^{n+1}_{i+0.5,j,k}}{\Delta z} - \frac{E_z\big|^{n+1}_{i+1,j,k+0.5}-E_z\big|^{n+1}_{i,j,k+0.5}}{\Delta x}\right) - \frac{1}{24}\left(\frac{E_x\big|^{n+1}_{i+0.5,j,k+1.5}-E_x\big|^{n+1}_{i+0.5,j,k+0.5}}{\Delta z} - \frac{E_z\big|^{n+1}_{i+1.5,j,k+0.5}-E_z\big|^{n+1}_{i+0.5,j,k+0.5}}{\Delta x}\right))$$

$$\text{C1By}=\left(\frac{2\varepsilon k_z-\sigma_z\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right)\ \text{C2By}=\left(\frac{2\varepsilon\Delta t}{2\varepsilon k_z+\sigma_z\Delta t}\right)$$

Bz update equation

$$B_z\Big|_{i+0.5,j,k+0.5}^{n+1.5} = \text{C1Bz}.\ B_z\Big|_{i+0.5,j+0.5,k}^{n+0.5} - \text{C2Bz}.\Big(\frac{9}{8}\Big(\frac{E_y\big|_{i+1,j+0.5,k}^{n+1} - E_y\big|_{i,j+0.5,k}^{n+1}}{\Delta x} - \frac{E_x\big|_{i+0.5,j+1,k}^{n+1} - E_x\big|_{i+0.5,j,k}^{n+1}}{\Delta y}\Big) - \frac{1}{24}\Big(\frac{E_y\big|_{i+1.5,j+0.5,k}^{n+1} - E_y\big|_{i+0.5,j+0.5,k}^{n+1}}{\Delta x} - \frac{E_x\big|_{i+0.5,j+1.5,k}^{n+1} - E_x\big|_{i+0.5,j+0.5,k}^{n+1}}{\Delta y}\Big)\Big)$$

$$\text{C1Bz}=\Big(\frac{2\varepsilon k_x - \sigma_x \Delta t}{2\varepsilon k_x + \sigma_x \Delta t}\Big)\ \text{C2Bz}=\Big(\frac{2\varepsilon \Delta t}{2\varepsilon k_x + \sigma_x \Delta t}\Big)$$

Hx update equation

$$H_x\Big|_{i,j+0.5,k+0.5}^{n+1.5} = \text{C1Hx}.\ H_x\Big|_{i+0.5,j+0.5,k}^{n+0.5} + \text{C2Hx.Bx}\Big|_{i,j+0.5,k+0.5}^{n+1.5} - \text{C3Hx.Bx}\Big|_{i,j+0.5,k+0.5}^{n+0.5}$$

$$\text{C1Hx}=\Big(\frac{2\varepsilon k_z - \sigma_z \Delta t}{2\varepsilon k_z + \sigma_z \Delta t}\Big)\quad \text{C2Hx}=\Big(\frac{2\varepsilon k_x + \sigma_x \Delta t}{\mu(2\varepsilon k_z + \sigma_z \Delta t)}\Big)$$

$$\text{C3Hx}=\Big(\frac{2\varepsilon k_x - \sigma_x \Delta t}{\mu(2\varepsilon k_z + \sigma_z \Delta t)}\Big)$$

Hy update equation

$$H_y\Big|_{i+0.5,j,k+0.5}^{n+1.5} = \text{C1Hy}.\ H_y\Big|_{i+0.5,j,k+0.5}^{n+0.5} + \text{C2Hy.By}\Big|_{i+0.5,j,k+0.5}^{n+1.5} - \text{C3Hy.By}\Big|_{i+0.5,j,k+0.5}^{n+0.5}$$

$$\text{C1Hy}=\Big(\frac{2\varepsilon k_x - \sigma_x \Delta t}{2\varepsilon k_x + \sigma_x \Delta t}\Big)\quad \text{C2Hy}=\Big(\frac{2\varepsilon k_y + \sigma_y \Delta t}{\mu(2\varepsilon k_x + \sigma_x \Delta t)}\Big)$$

$$\text{C3Hy}=\Big(\frac{2\varepsilon k_y - \sigma_y \Delta t}{\mu(2\varepsilon k_x + \sigma_x \Delta t)}\Big)$$

Hz update equation

$$H_z\Big|_{i+0.5,j+0.5,k}^{n+1.5} = \text{C1Hz}.\ H_z\Big|_{i+0.5,j+0.5,k}^{n+0.5} + \text{C2Hz.Bz}\Big|_{i+0.5,j+0.5,k}^{n+1.5} - \text{C3Hz.Bz}\Big|_{i+0.5,j+0.5,k}^{n+0.5}$$

$$\text{C1Hz}=\Big(\frac{2\varepsilon k_y - \sigma_y \Delta t}{2\varepsilon k_y + \sigma_y \Delta t}\Big)\quad \text{C2Hz}=\Big(\frac{2\varepsilon k_z + \sigma_z \Delta t}{\mu(2\varepsilon k_y + \sigma_y \Delta t)}\Big)$$

$$\text{C3Hz}=\Big(\frac{2\varepsilon k_z - \sigma_z \Delta t}{\mu(2\varepsilon k_y + \sigma_y \Delta t)}\Big)$$

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