Design and Performance Evaluation of a Quadrature Mirror Filter (QMF) Bank

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Abstract
In this paper, a two channel Quadrature Mirror Filter (QMF) bank is designed and the performance is evaluated. A new method is developed to optimize the prototype filter response. The design problem is formulated as nonlinear unconstrained optimization of an objective function, which is weighted sum of square of error in passband, stop band and overall filter bank response. The proposed method gives better performance in terms of peak reconstruction error and computational time.

Keywords
Filter Banks, Quadrature Mirror Filter (QMF), Subband Coding, Multirate Filter Banks

I. Introduction

Over the past two decades, the design of filter banks has received considerable attention in numerous fields such as speech coding, scrambling, image processing etc. [1]. Among the various filter banks, two-channel QMF bank was the first type of filter bank used in signal processing applications for separating signals into subbands and reconstructing them from individual subbands [2-3]. Subsequently, a substantial progress has been made in other fields like antenna systems [4], Analog to Digital (A/D) convertor [5], and design of wavelet base [6], due to advances in QMF bank. There are three types of distortions in QMF banks: aliasing distortion, phase distortion and amplitude distortion [1]. The aliasing distortion is eliminated with the use of suitable design of the synthesis filters and phase distortion is removed with the help of a linear phase FIR filter. Amplitude distortion can be minimized using computer aided techniques or can be equalized by cascading with a filter. Johnston [7], has introduced the concept of a two band linear phase QMF banks. In linear phase QMF banks, aliasing and phase distortions are removed by choosing the analysis /synthesis filters and are assumed to have linear phase with even length. A nonlinear and iterative approach is used to minimize the amplitude distortion. The objective function is formulated using weighted sum of the ripples in the system response.

II. Two-Channel Filter Banks

The two-channel filter bank is shown in fig. 1, [8]. The input signal x[n] is splitted into two subband signals having equal bandwidth, using lowpass and highpass analysis filters H_0 (Z) and H_1 (Z) respectively. These subband signals are down sampled by a factor of two to achieve signal compression or to reduce complexity. At output side, the two subband signals are interpolated by a factor of two and passed through lowpass and highpass synthesis filters F_0 (Z) and F_1 (Z), respectively.

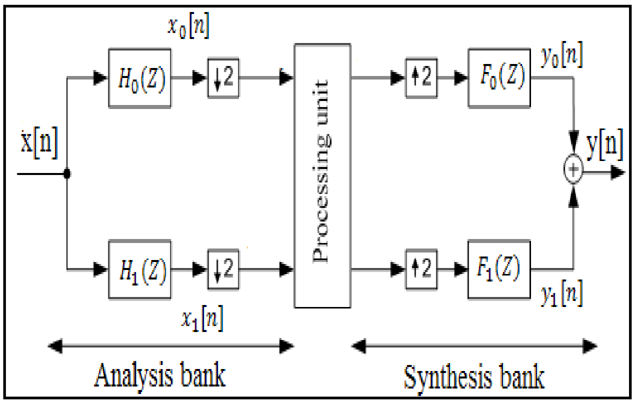

Fig. 1: Two-Channel Analysis/Synthesis Filter Bank

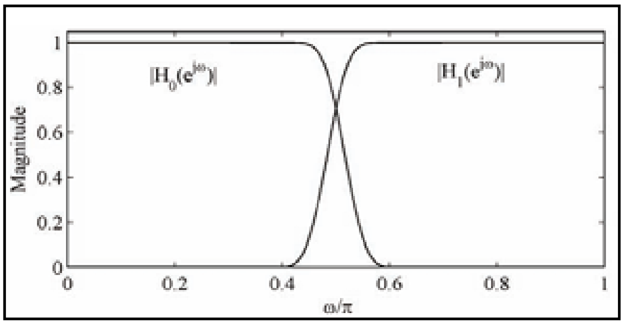

Fig. 2: Typical Magnitude Responses of the Lowpass Filter, and the Highpass Filter [8]

The reconstructed signal y[n] is different from the input signal x[n] due to three errors: aliasing distortion, amplitude distortion and phase distortion. To obtain a Perfect Reconstruction (PR) or near perfect reconstruction (NPR) these three distortions are eliminated or minimized using

- Alias-free filter bank
- Perfect reconstruction filter bank

Fig. 2 [8], shows the filter response of the lowpass filter H_0 (Z) and highpass filter H_1 (Z). The z-Transforms of the input signals are

$X_0(z) = X(z)\, H_0(z)$

$X_1(z) = X(z)\, H_1(z)$ (1)

The output signals $y_0[n]$ and $y_1[n]$ are added to obtain the single output y[n]. The z-transform of y[n] is

$Y(Z) = Y_0(Z) + Y_1(Z)$

$$Y(Z) = \frac{1}{2}[H_0(Z)F_0(Z) + H_1(Z)F_1(Z)]X(Z)$$

$$+\frac{1}{2}[H_0(-Z)F_0(Z) + H_1(-Z)F_1(Z)]X(-Z)$$

$Y(Z) = T(Z)\, X(Z) + A(Z)\, X(-Z)$ (2)

where, T(Z) is the distortion transfer function and A(Z) is the aliasing distortion.

A. Alias-Free Filter Banks

The alias-free design of the two-channel filter bank means the elimination of the A(Z) aliasing distortion in equation (2).

where, $T(z)=\frac{1}{2}[H_0(Z)F_0(Z) + H_1(Z)F_1(Z)]$is the overall distortion transfer function and

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$A(z) = \frac{1}{2}[H_0(-Z)F_0(Z) + H_1(-Z)F_1(Z)]$ is the aliasing transfer function [9].
In order to obtain an alias-free filter bank, A(z) has to be cancelled, that is, A(z) = 0. To avoid aliasing distortion setting, $F_0(Z) = H_1(-Z)$ and $F_1(Z) = -H_0(-Z)$.
Now, Y(Z) = T(Z)X(Z) where,
$T(Z) = H_0(Z)H_1(-Z) - H_0(-Z)H_1(Z)$, and T(z) is called the distortion transfer function. In the frequency domain, put $z = e^{j\omega}$

$$T(e^{j\omega}) = |T(e^{j\omega})|e^{j arg[T(e^{j\omega})]} \quad (3)$$

where, $|T(e^{j\omega})|$ represents the amplitude distortion, and $arg[T(e^{j\omega})]$ is the phase distortion of the analysis/synthesis filter bank.

B. Perfect-Reconstruction Filter Banks

The perfect-reconstruction property means that the signal at the output of the analysis/synthesis filter bank y[n] is a delayed version of the original signal x[n], i.e,

$$Y[n] = x[n-k] \quad (4)$$

More economical solutions with the reduced computational complexity can be obtained if the filter bank achieves the nearly perfect-reconstruction property defined by

$$y[n] \approx x[n-k] \quad (5)$$

The distortion transfer function of the two-channel analysis/synthesis filter bank satisfying the perfect reconstruction property is a pure delay, $T(z) = Z^{-n0}$

$$\arg[T(e^{j\omega})] = K\omega \quad (6)$$

For the nearly-perfect reconstruction property, the distortion transfer function is an approximation of the ideal solution of equation (4). If T(z) is an all-pass transfer function,

$$|T(e^{j\omega})| = \text{Constant for all } \omega \quad (7)$$

In this case, T(z) is a linear phase transfer function, i.e., there are no phase distortions in the analysis/synthesis filter bank.

III. Design of a Quadrature Mirror Filter (QMF) Bank

Two-channel QMF bank was the first type of filter bank used in signal processing applications for separating signals into sub bands and reconstructing them from individual sub bands.
The term Quadrature Mirror (QMF) Filter bank [10], denotes the quadrature mirror symmetry of the lowpass/highpass filter pair $H_0(z)$ and $H_1(z)$.
In equation (2), T(Z) is the distortion transfer function and A(Z) is the aliasing distortion, which completely eliminated with use of the condition given below,

$H_1(z) = H_0(-Z)$, $F_0(Z) = 2H_1(-Z)$ and
$F_1(Z) = -2H_0(-Z)$.

Transfer function T(Z) is given below,

$$T(Z) = H_0^2(Z) - H_1^2(Z)$$
$$= H_0^2(Z) - H_0^2(-Z) \quad (8)$$

The amplitude and phase distortions of the overall QMF bank depend on the performances of the lowpass filter $H_0(z)$. If $H_0(z)$ has a linear phase, the phase distortion of the overall filter bank is eliminated. It is shown that for $H_0(z)$ being a linear-phase FIR filter of the length N, the overall frequency response $T(e^{j\omega})$ of the QMF bank can be written as

$$T(e^{j\omega}) = e^{-j\omega(N-1)}[|H_0(e^{j\omega})|^2 - (-1)^{(N-1)}|H_1(e^{j\omega})|^2] \quad (9)$$

Since the filter pair $[H_0(z), H_1(z)]$, is a halfband filter pair, their magnitude responses at the crossover frequency $\omega_c = \pi/2$ are equal, i.e., $|H_0(e^{j\pi/2})| = |H_1(e^{j\pi/2})|$. This implies that for odd values of N, $T(e^{j\omega})$ may have severe amplitude distortions in the vicinity of $\omega = \pi/2$. Therefore, when using the linear-phase FIR filters for a QMF bank, the filter length N should be an even number. When the filter length N is an even number, equation (10) [11] reduces to

$$T(e^{j\omega}) = e^{-j\omega(N-1)}[|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2] \quad (10)$$

It follows from the above equation that the FIR QMF filter bank with linear-phase filters in the analysis and synthesis parts would satisfy the condition of a perfect reconstruction if

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 = 1 \quad (11)$$

There by, the perfect reconstruction may be achieved when the linear-phase analysis filters are power complementary. The QMF bank with linear phase filters has no phase distortion, but the amplitude distortion [11], will always exist. Hence,

$$|H_0(e^{j\omega})|^2 + |H_1(e^{j\omega})|^2 \cong 1 \quad (12)$$

If the prototype filter is assumed ideal in passband and stopband, then there is reconstruction error in transition region, which can be minimized for improving the performance of the filter bank.

$$PER = max20log_{10}\{|H_0(e^{j\omega})|^2 + |H_0(e^{j(\omega-\pi)})|^2\} \quad (13)$$

The objective function $(\emptyset)$ is constructed using weighted sum of mean square of errors in these responses. Now, the objective function $\emptyset$ to be minimized can be chosen as a linear combination of two functions: (1) Stopband attenuation of $H_0(z)$ and (2) The sum of the square-magnitude responses of $H_0(z)$ and $H_1(z)$ as shown in the equation (12). The objective function is given by

$$\emptyset = max\left\{|H_0(e^{j\omega})|^2 + |H_1(e^{j(\omega-\frac{\pi}{2})})|^2 - 1\right\} \quad (14)$$

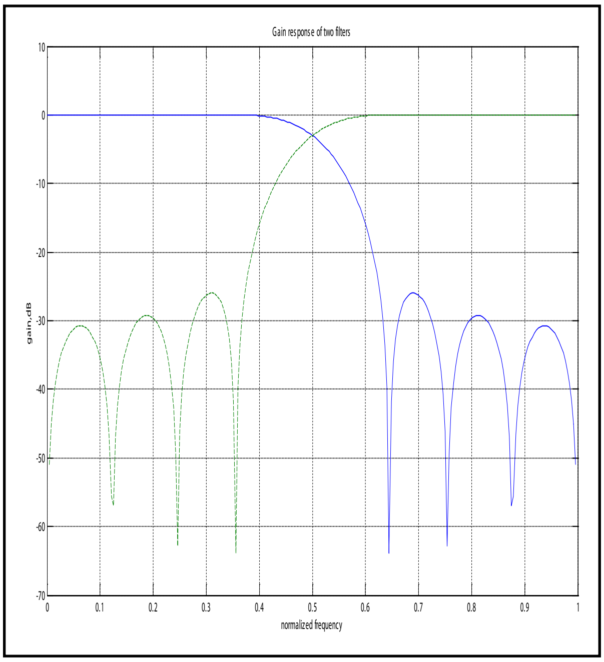
The objective function $\emptyset$ has been made very small by the minimization procedure [12]. This will make $H_0(z)$ have a magnitude response satisfying $|H_0(e^{j\omega})| \cong 1$ in its passband and $|H_0(e^{j\omega})| \cong 0$ in its stopband. Moreover, since the power-complementary condition of equation (12) will be satisfied approximately, the magnitude response of the power-complementary highpass filter $H_1(z)$ to the lowpass filter will satisfy $|H_1(e^{j\omega})| \cong 0$ in the passband of $H_0(z)$ and $|H_1(e^{j\omega})| \cong 1$ in the stopband.

IV. Results and Discussion

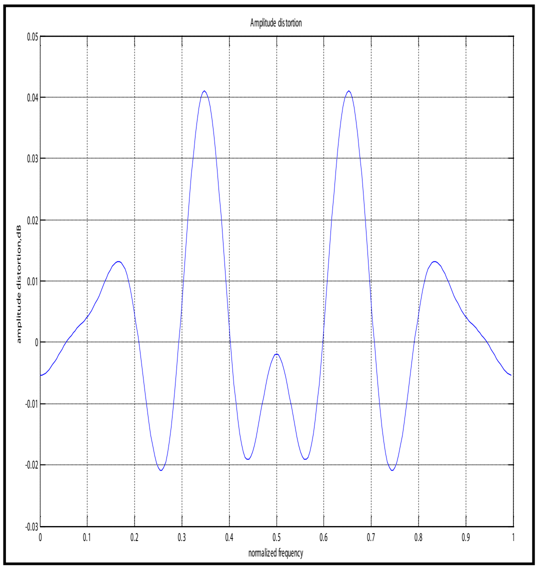
The performance of the designed filter are evaluated in terms of Peak Reconstruction Error (PRE) and stopband edge attenuation. The simulations are carried out using MATLAB.
In Table 1, different values of filter coefficient used for different filter taps (N) are shown. Fig. 3(a) and Fig. 4 (a), show the amplitude response of QMF bank for different values of filter lengths. The variation of reconstruction error is depicted in fig. 3(b) and fig. 4 (b). If the filter length increases amplitude distortion reduces and computational time also reduces and gradually become negligible. Table 2, shows the Peak Reconstruction Error (PRE) and amplitude distortion for different filter lengths.

Table 1: Coefficients of the Lowpass Filter in Different Examples

	Filter Taps	Filter Taps	Filter Taps	Filter Taps
Length (N)	16	22	24	32
Coefficients	B1	B1	B1	B1
0	0.009095	0.000656	0.002964	0.001655
1	-0.024 191	-0.003 535	-0.006 208	-0.002 954
2	0.003 421	0.008 654	-0.002 532	-0.001 490
3	0.049 317	0.003 001	0.015 046	0.006 213
4	-0.029 367	-0.024 836	-0.001 278	0.000 771
5	-0.100 047	0.006 987	-0.028 997	-0.011 798
6	0.120 719	0.048 472	0.011 369	0.001 252
7	0.470 753	-0.031 247	0.051 950	0.020 213
8	-	-0.098 388	-0.036 576	-0.006 163
9	-	0.120 155	-0.099 753	-0.032 343
10	-	0.470 865	0.126 108	0.016 933
11	-	-	0.467 839	0.053 913
12	-	-	-	-0.041 756
13	-	-	-	-0.099 830
14	-	-	-	0.130 903
15	-	-	-	0.465 027

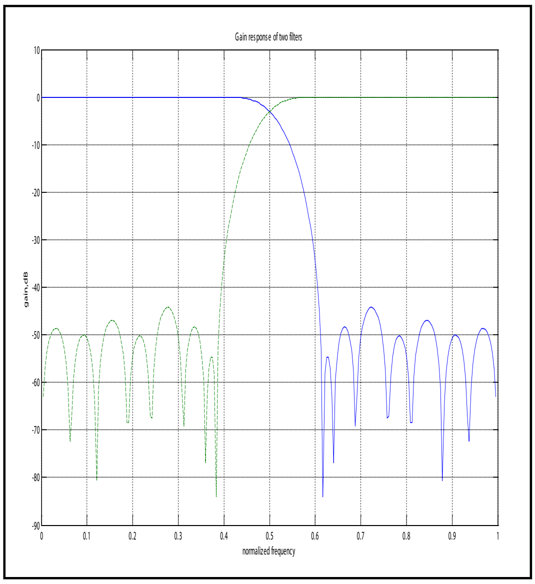


(a)

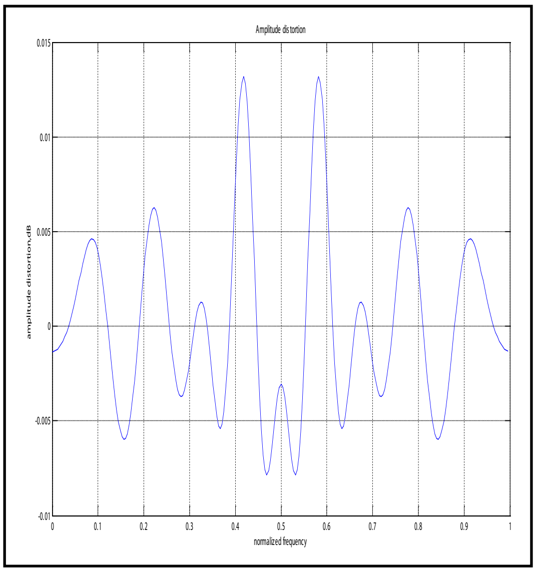


(b)

Fig. 3: Two-Channel FIR QMF Bank, (a). Gain Response (b). Amplitude Distortion for (N=16)



(a)



(b)

Fig. 4: Two-Channel FIR QMF Bank, (a). Gain Response, (b). Amplitude Distortion for (N=32)

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Table 2: Summary of Results

Filter Taps	PRE	Amplitude Distortion
Length(N)	(In dB)	eam(max)
16	0.0410	0.0048
22	0.0237	0.0044
24	0.0205	0.0038
32	0.0132	0.0018

Simulation results show that the proposed method leads to filter banks with improved performance in terms of amplitude distortion and computation time required.

V. Conclusion

In this paper, a new objective function has been proposed for the design of Quadrature Mirror Filter (QMF) bank. The proposed method optimizes the prototype filter response characteristics in passband, stopband and also the overall filter bank response. Simulation results show improved performance with regard to amplitude distortion and complexity and also decrease the peak reconstruction error. The future scope includes applications in subband coding for speech and image signals and transmultiplexers for telecommunications.

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