Frequency Transformation in Bilinear Digital IIR Filter by Using Pascal Matrix

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Abstract
The transfer function $H(f)$ can be calculated from the matrix operation of frequency transformation and the “Pascal matrix” transformation. Here the proposed method is a fractional technique which is used to obtain low pass filter and high pass filter transfer function from another function by the above said method, to it fractional $z$ is introduced. The proposed method of transfer function given better results for different fractions of $z$ integer of side lobe attenuation than the existing $z$ transfer function.

Keywords
Frequency Transformation, Pascal Matrix, Bilinear Digital IIR Filter

I. Introduction
A filter is linear time invariant system used for removing undesirable noise from desired signal. It can be used in spectral shaping such as equalization of communication channels, signal detection in radar, sonar etc. A filter is designed to pass a band of desired frequencies without any distortion called pass band of the filter and to totally block a band of unwanted frequencies called stop band of the filter. The digital filters are available as low pass and high pass filters. A low pass filter blocks all frequencies above the specified cut off frequency similarly high pass filter passes all frequencies above the specified cut off frequency. Here Pascal matrix is used for transforming normalized analog transfer function $H(s)$ from the lowpass to the high pass discrete waveform $H(z)$. In this paper we used fractional $z$ inverse values to find the frequency response of the low pass and high pass filters (Infinite Impulse Response), and it is found that the responses are improved when compared with integer $z$ inverse values.

II. IIR Filters
The most IIR (Infinite Impulse Response) filters are unstable but having limited cycles and these can be derived from analog form. The IIR filters are available as, Butterworth and Chebyshev filters. The IIR filters can be designed in two ways, viz bilinear transformation and Impulse Invariance method. The Bilinear transformation is implemented as follows:

Step-1: From the given specifications find prewarping analog frequencies using formula

$$\Omega = \frac{\pi \tan \left(\frac{\pi}{2}\right)}{s}$$

Step-2: Using the analog frequencies find $H(s)$ of the analog filter.
Step-3: Select the sampling rate of digital filter call it $T$ in seconds per sample.
Step-4: Substitute $s = \frac{2(1 - z^{-1})}{T(1 + z^{-1})}$ into the transfer function in step 2.

Considering the low pass transfer function $H(s)$ to low pass and high pass transfer function $H(z)$ using the following features specified by $C=1$, $f_s=8000$Hz

$$H(s) = \frac{s^2 + 5.153}{0.929s^2 + 2.781s^2 + 4.344s + 5.153}$$

Low pass transformation from s domain to z domain. By following the above steps we can use Bilinear Transformation for the given specification the transfer function can be derived in z-domain.

$$H(z) = \frac{0.4658 + 1.0948z^{-1} + 1.0948z^{-2} + 0.4658z^{-3}}{1 + 1.0778z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}}$$

Similarly transformation of low pass to high pass from s domain to z domain yields the $f$ transfer function as below,

$$H(z) = \frac{0.4658 - 1.0948z^{-1} + 1.0948z^{-2} - 0.4658z^{-3}}{1 - 1.0778z^{-1} + 0.8420z^{-2} - 0.20149z^{-3}}$$

III. Proposed Fractional -Z Method

The transfer function of (1) a low pass filter after applying the fractional $z$ method becomes as follows.

For $z^{\alpha} = (1 - \alpha) + \alpha z^{-1}$

For $z^{\alpha3} = 0.9 + 0.1z^{-1}$

$$H(z) = \frac{0.4658 + 1.0948z^{-1} + 1.0948z^{-2} + 0.4658z^{-3}}{1.97002 + 0.10778z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}}$$

From the above waveforms we observed that the RSA (Relative Side lobe Attenuation) for low pass and high pass filter is -15.2dB.

Fig. 1: Frequency Response of Low Pass Filter for $z^{-1}$

Fig. 2: Frequency Response of High Pass Filter for $z^{-1}$

Frequency response of a high pass filter without fractional $z$ inverse values

From the above waveforms we observed that the RSA (Relative Side lobe Attenuation) for low pass and high pass filter is -15.2dB.
For $z^{-0.2} = 0.8 + 0.2z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.9700 + 0.1077z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]
For $z^{-0.3} = 0.7 + 0.3z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.7544 + 0.3233z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]
For $z^{-0.4} = 0.6 + 0.4z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.6466 + 0.4311z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]
For $z^{-0.5} = 0.5 + 0.5z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.5389 + 1.5389z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]
For $z^{-0.6} = 0.4 + 0.6z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.4311 + 0.6468z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]
For $z^{-0.7} = 0.3 + 0.7z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.3233 + 0.7544z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]
For $z^{-0.8} = 0.2 + 0.8z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.2156 + 0.8622z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]
For $z^{-0.9} = 0.1 + 0.9z^{-1}$
\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.1077 + 0.9700z^{-1} + 0.8420z^{-2} + 0.2015z^{-3}} \]

### IV. Result Analysis for Low Pass Filter

The frequency response for $z^{-0.1}$ is

![Fig. 3: Frequency Response of Low Pass Filter for $z^{-0.1}$](image)

The frequency response for $z^{-0.2}$ is

![Fig. 4: Frequency Response of Low Pass Filter for $z^{-0.2}$](image)

### Table 1: For Low Pass Filter

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>Relative Side Lobe Attenuation (dB)</th>
<th>Bandwidth (dB)</th>
</tr>
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<tr>
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<td>-15.2</td>
<td>0.14062</td>
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<tr>
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<td>-15.8</td>
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<tr>
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<td>0.8</td>
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</tr>
<tr>
<td>0.9</td>
<td>-14.5</td>
<td>0.14062</td>
</tr>
</tbody>
</table>

The transfer function of (2) a high pass filter after applying the fractional $z$ method becomes as follows.

For $z^{0.1} = (1-\alpha) + \alpha z^{-1}$

For $z^{0.2} = 0.9 + 0.1z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.0305 + 1.0772z^{-1} + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{0.3} = 0.8 + 0.2z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{1.01304 + 1.21544z^{-1} + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{0.4} = 0.7 + 0.3z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{0.24565 + 0.32315z^{-1} + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{0.5} = 0.6 + 0.4z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{0.3538 + 0.4308z^{-1} + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{0.6} = 0.5 + 0.5z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{0.4615 + 0.5385z^{-1} + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{0.7} = 0.4 + 0.6z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{0.5692 + 0.6462 + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{0.8} = 0.3 + 0.7z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{0.6592 + 0.7642 + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{0.9} = 0.2 + 0.8z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{0.7892 + 0.8692 + 0.8420z^{-2} + 0.20149z^{-3}} \]

For $z^{1.0} = 0.1 + 0.9z^{-1}$

\[ H(z) = \frac{0.4658 + 1.0948z^{-1} + 0.8420z^{-2} + 0.4658z^{-3}}{0.8923 + 0.9693z^{-1} + 0.8420z^{-2} + 0.20149z^{-3}} \]
For \( z^{-0.9} = 0.1 + 0.1z^{-1} \)
\[
H(z) = \frac{0.4658 + 1.0948z^{-1} + 1.0948z^{-2} + 0.4658z^{-3}}{0.9794 - 1.00554z^{-1} + 0.08420z^{-2} - 0.20149z^{-3}}
\]

V. Result Analysis for High Pass Filter
The frequency response of high pass filter for \( z^{-0.1} \) is

![Fig. 6: Frequency Response of Low Pass Filter for \( z^{-0.1} \)](image)
The frequency response of high pass filter for \( z^{-0.2} \) is

![Fig. 7: Frequency Response of Low Pass Filter for \( z^{-0.2} \)](image)
The frequency response of high pass filter for \( z^{-0.3} \) is

![Fig. 8: Frequency Response of Low Pass Filter for \( z^{-0.3} \)](image)

Table 2: For High Pass Filter

<table>
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VI. Conclusion
From the proposed method of fractional transfer function, we noticed that it is tunable for some fractional of \( z \) getting better result than the fixed \( z \) transfer function as shown in the Table 1 & 2 for the low pass and high pass transfer functions.

References