Impact of Input Frequency and Modulator Order on the Performance of Sample Rate Converter

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Abstract
The sample-rate converter introduces a performance degradation. The performance degradation depends on number of variables.These variables are the output sampling frequency, the conversion factor and the input frequency of sine wave. Furthermore the order of the sigma-delta modulator is a variable when we consider the signal to noise ratio of the output signal of the sample rate converter. In this paper, the dependence of the SNR will be investigated as function of the input frequency for a second-order, a third order and a fourth order sigma-delta modulator.

Keywords
Sample Rate Converter, Spectral Analysis, Upsample-Downsample Filter, Sigma-Delta Modulator, Frequency Detector.

I. Introduction

The increasing need in modern digital systems to process data at more than one sampling rate has led to the development of new sub-area in DSP known as multirate processing [1]. The two primary operations in multirate processing are decimation and interpolation and they enable the data rate to be altered in an efficient manner. Decimation reduces the sampling rate (that is, the sampling frequency), effectively compressing the data and retaining only the desired information. Interpolation on the other hand increases the sampling rate. Often the purpose of converting data to a new rate is to make it easier (for example, computationally more efficient) to process or to achieve compatibility with another system. This process is necessary in different situations: Digital Audio Workstation (DAW) users often record and edit at a high sample rate, and then down-sample the audio to get it onto various media. This sample rate conversion can either be done by the DAW during or after the bounce, or in a separate application after bouncing. In another scenario, sample rate conversion is necessary when audio material recorded for a specific media (e.g. CD) gets transferred to a different media (e.g. DVD, DAT or Digital Video). For example, a DVD audio project requires sample rate conversion from 96 kHz to 44.1kHz in order to be transferred to CD, and a CD audio project requires conversion from 44.1 kHz to 48 kHz to be transferred to Digital Video format.

It is very important for the sample rate conversion to be as transparent as possible. Ideally, when converting from an original into a new sample rate, we would like the converted signal fidelity to be as high as if we had directly sampled it from the original analog signal. This degree of perfect transparency is possible only in theory, since we would need a computer with infinite memory and infinite processing power to achieve it. In practice, however, a very high degree of transparency can be achieved with a high-quality sample rate converter.

In this paper, analysis results are shown for a new method of digital sample rate converter [2]. The operation principle of the new method of sample rate conversion is very simple. An input sample is directly transferred to the output, while per unit of time, a certain amount of these samples is omitted or repeated, depending on the difference in input and output sample frequencies. The omission, acceptance or repetition of a sample is called 'validation'. In order to get the simplest hardware implementation, the choice has been made to use only the take-over operation and the repetition operation in the current system solution. This means that the output sampling frequency of the sample rate converter is always larger than the input sample frequency.

The process of repeating samples inevitably introduces errors. The resulting output samples will have correct values, but as a result of the validation operation, they are placed on the output time grid with a variable time delay with respect to the input time grid. As a consequence, the output sequence should be viewed as the input sequence, having the correct signal amplitude, which is sampled at wrong time moments. The effect is the same as sampling the input signal by a jittered clock [3]. As a result, it can be stated that the time error mechanism introduced by the validation algorithm is time jitter.

If all input samples would be transferred to the output grid without the repetition or omission of a certain amount of them, then the output signal would be just a delayed version of the input signal, exhibiting the same shape. It is the repetition and omission (in the current system setup only the repetition) of input samples that give rise to a variation in time delay for each individual output sample. This variation in individual time delays introduces phase errors. As a result of this, the shape of the output signal will be distorted [4].

The time errors introduced by the conversion process can be reduced considerably by applying upsampling and downsampling techniques. The input sample rate of the converter will be higher so that the conversion errors are smaller, resulting in smaller time jitter. These techniques do not suffice when we want to achieve the very high analog audio performance required for professional applications [5]. By using a sigma-delta modulator (noise shaper) as control source for the conversion process, the time errors will be shaped to the higher frequency region. As a result, the audio quality (in the baseband) of the signal will be preserved, provided that enough bandwidth is created by upsampling of the input signal. The high frequency (out of base band) phase modulation terms can be filtered by a decimation filter or an analog low-pass filter which is directly placed after the sample-rate converter [6,7]. Fig. 1 shows the block diagram of the complete sample-rate converter.

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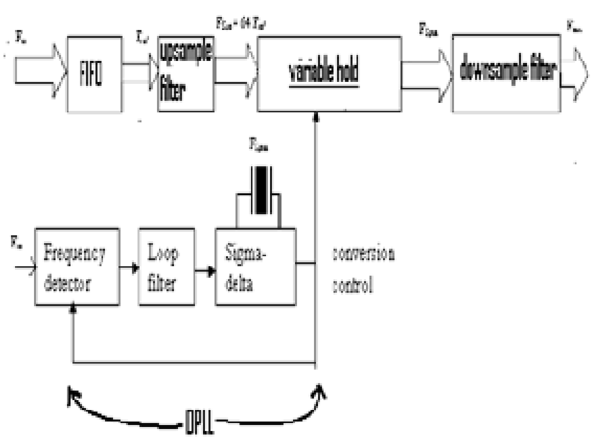

Fig. 1: Block diagram of the sample-rate converter.

As has already been mentioned, only the input sample take over operation will be employed here in order to get the simplest hardware. This means that the input sample frequency of the converter must be always be smaller than the output sample frequency. With this restriction imposed, it is assured that all input samples are used in the output sequence, none of them being omitted. The extra output samples per unit of time are inserted in the output sequence by repetition of their previous output samples.

II. Sample Rate Converter Properties

In this part, the properties of the proposed sample-rate converter in the frequency domain will be investigated. It is observed that the first order approximation of the amplitude error is accurate enough, even for the worst case situation. The continuous-time description of the first-order model is:

$$y(t) = x(t) - \dot{x}(t)\,\Delta t(t) \qquad (1)$$

Fig. 2 gives a block diagram of this first-order model.

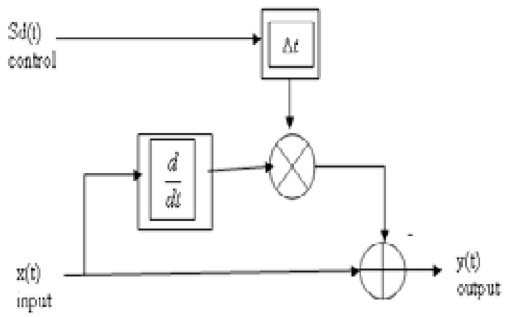

Fig. 2: Block diagram of the first-order model.

When we want to convert (1) to discrete-time, we have to keep in mind that the output samples have a different sample time than the input samples. We will therefore enter two discrete-time variables; $k.T_{S,out}$ for the output samples and time delays, and $l.T_{S,in}$ for the input samples (shortly denoted by k and l). The discrete-time model now becomes:

$$y(k) = x(l) - \dot{x}(l)\,\Delta t(k) \qquad (2)$$

Normally the derivative of a discrete-time signal is determined by the amplitude difference of the current sample and its previous sample, divided by the sample time. In our case it is more correct to use the difference between the next input sample and the present input sample, because the position in time of the present output sample is between the time moments of those two input samples. For the time derivative of the input signal x(l) we obtain:

$$\dot{x}(l) = \frac{x(l+1) - x(l)}{T_{S,in}} = F_{S,in}.[x(l+1) - x(l)] \qquad (3)$$

Substituting this into (2), we get

$$y(k) = x(l) - F_{S,in}.[x(l+1) - x(l)].\Delta t(k) \qquad (4)$$

In order to become known with the spectral density of the output signal y(k), we must firstly determine the correlation function:

$$R_{yy}(n) = E\{y(k).y(k+n)\} \qquad (5)$$

This correlation function describes the correlation between the present output sample (k_0) and the (k_0+n)-th output sample and is therefore dependent on the output sample time $T_{S,out}$. Note that n must be an integer.

The problem arises that for a time step of n samples (=$n.T_{S,out}$) in the output signal we must know the corresponding time step in the input signal. Assume that this time step is equal to $m.T_{S,in}$, that is, y(k+n) corresponds to x(l+m). The relation between m and n then becomes:

$$n.T_{S,out} = m.T_{S,in} \Rightarrow m = \frac{T_{S,out}}{T_{S,in}}.n \Rightarrow m = \frac{F_{S,in}}{F_{S,out}}.n \qquad (6)$$

The conversion factor for the sample-rate conversion process is not necessarily a rational number, which implies that m is not necessarily an integer. For the calculation of the discrete-time correlation function we need both n and m, as we have two discrete-time variables. The problem is that the discrete-time input signal x(l+m) is not defined when m is not an integer. We must therefore conclude that the correlation function $R_{yy}(n)$ of the discrete-time output signal can not be solved analytically.

Consider the discrete-time description of the first-order model (2). For the calculation of an output sample on time moment k (somewhere between l and l+1) the discrete-time derivative of the input signal x(l) on time moment k is needed. This derivative is determined using the two adjacent input samples (4). Suppose x(t) is the continuous-time signal constructed out of the input samples x(l) using linear interpolation. The continuous-time derivative of this input signal is in fact similar to the discrete-time derivative given by (3). In fact we deduce our discrete-time analysis from the continuous-time analysis. In order to find out the spectral properties of the sample-rate converter, it is therefore allowed that we use the continous-time description given by (1).

III. Simulation and Performance Analysis

The sample-rate converter has been simulated with a second-order, a third order and a fourth order sigma-delta modulator using a fixed conversion factor of 1.684 and an output frequency of 128 FS . The input frequency is varied from 1kHz to 20kHz. Fig. 3 shows the simulated SNR as function of the input frequency for the three sigma delta modulators.

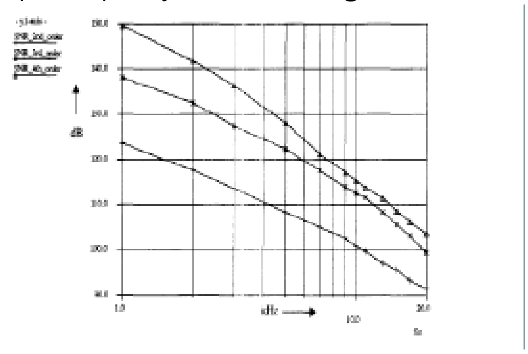

Fig. 3: The SNR as function of the frequency of the input sinewave for a second-order, a third order and a fourth order sigma-delta modulator.

The simulation results correspond to the expectations. The slope is low for low frequencies, equal to about -20 dB per decade. For higher frequencies, the SNR decreases faster than 20 dB per decade, due to increase of quantization noise in the audio baseband. The minimum SNR of the output sinewave is for the three sigma-delta modulators 92, 100 and 104 dB (second-, third and fourth –order respectively). For this conversion factor (1.684) and 16-bit performance is satisfied for the third-order and fourth-order sigma-delta modulator only.The performance of the three sigma-delta modulators is explained in following subparagraphs.

A. Control by a second-order sigma-delta modulator

The difference in SNR for 1kHz and 20 kHz is theoretically predictable. The amplitude of error power is dependent on the square of the input frequency, which yields a slope of -20 dB per decade. The difference between 1kHz and 20kHz correspond to one decade and one octave on the frequency axis. The difference in SNR between 1kHz and 20kHz is, due to the amplitude of error power, 20+6=26dB. A second order sigma-delta modulator implies that the time error spectrum has a first order slope. The frequency shift of this time error spectrum causes another loss of 6dB in SNR when the 1kHz and 20 kHz input frequencies are compared. Theoretically the difference in SNR between 1kHz and 20kHz will be 32dB where a difference of 32.17 dB is measured from the simulation results (Fig. 3).

B. Control by a Third Order Sigma-Delta Modulator

The difference in SNR between the 1kHz and 20kHz input signal is 26 dB when only the amplitude of the error power is accounted for. The additional difference is determined by the order of the sigma-delta modulator. The time error spectra is second order in this case, so the additional difference between 1kHz and 20kHz is 2*6=12 dB. Theoretically, the total difference between the 1kHz and 20kHz input amounts 26+12=38 dB, where a difference of 38.81 is measured from the simulation results (Fig. 3).

C. Control by a Fourth Order Sigma-Delta Modulator

The difference in SNR between the 1kHz and 20 kHz input signal is 26 dB when only the amplitude of the error power is accounted for. The sigma-delta modulator is fourth order, which implies that the time error is third order. Another 3*6=18 dB of difference is expected. Theoretically, the difference in SNR between a 1kHz and 20 kHz input signal is 26+18=44 dB, whereas the simulation results show a difference of 45.97dB.

IV. Conclusion

It is concluded that the increase in SNR between a second -order and a third-order sigma-delta modulator is larger than between a third-order and a fourth-order sigma-delta modulator, especially for high frequencies. For a higher-order sigma-delta modulator, the amount of quantization noise shifted in the baseband is larger because the slope of the spectrum is larger. It appears that there is limit for the achievable SNR when a higher-order sigma-delta modulator is used. This is due to the fact that the amount of quantization noise shaped out of the baseband is limited.

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