Performance evaluation of Butterworth Filter for Signal Denoising

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Abstract
A signal in the communication system is the information containing part which is to be process, but during the processing of the signal some noise is added in the signal and signal becomes noisy. This is now mandatory to eliminate this noise from the signal to get information from the signal. In this paper, a wavelet filter based on Butterworth IIR filter is designed for the signal analysis. Butterworth low pass IIR filter has limited application in signal denoising. It has maximally flat response in the pass band and, therefore, the filter has distorted output. It makes Butterworth filter little applicable in communication systems and other signal analysis technique. The newly designed matched wavelet filter presents a new concept for better signal analysis and disturbance detection in the communication systems. The limitation of Butterworth low pass filter in signal denoising and other applications can be eliminated by using this matched wavelet filter. We have improved performance in signal filtering by using Butterworth wavelet filter. This wavelet filter finds applications in signal analysis, communication system and image compression with a lot of other fields.

I. Definition of wavelet

Wavelets are mathematical functions that cut up data into different frequency components, and then study each component with a resolution matched to its scale. They have advantages over traditional Fourier methods in analyzing the situations where the signal contains discontinuities and sharp spikes.

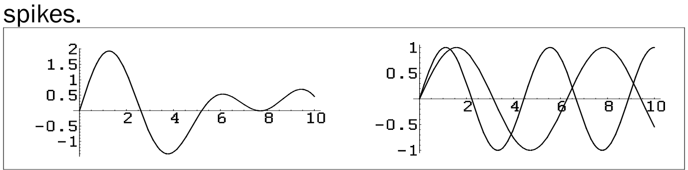

Fig. 1: Signal representation with different frequencies at a time instant.

Wavelets were developed independently in the field of the quantum physics, electrical engineering, and seismic geology. Interchanges between these fields during the last ten years have led to many new wavelet applications such as image compression, turbulence, human vision, radar, and earthquake prediction. The fundamental idea behind wavelets is to analyze according to scale. Indeed, some researchers in the wavelet field feel that, by using wavelets, one is adopting a whole new mindset or perspective in processing data. Wavelets [1] are functions that satisfy This idea is not new. Approximation using superposition of functions has existed since the early 1800's, when Joseph Fourier discovered that he could superpose sines and cosines to represent other functions. However, in wavelet certain mathematical requirements and are used in representing data or other functions. analysis, the scale that we use to look at data plays a special role.

II. Introduction to Filters

Filters are the electronic circuits which have the functionality to remove the unwanted frequency components from the signal. In the field of the digital signal processing, filters have the numerous applications. In signal processing, the function of a filter is to remove unwanted parts of the signal, such as random noise, or to extract useful parts of the signal, such as the components lying within a certain frequency range. Filters have functions of the signal separation and signal restoration [8]. The signal separation is done when signal is affected by the interference, noise or by other noise factors. For example, imagine a device for measuring the electrical activity of a baby's heart while still in the womb. The raw signal will likely be corrupted by the breathing and heartbeat of the mother. A filter might be used to separate these signals so that they can be individually analyzed. Signal restoration is used when a signal has been distorted in some way. For example, an audio recording made with poor equipment may be filtered to better represent the sound as it actually occurred. Another example is the deblurring of an image acquired with an improperly focused lens, or a shaky camera.

A. Introduction to Butterworth filter and Calculation of order and coefficients

The Butterworth filter is one type of signal processing filter design. It is designed to have a frequency response which is as flat as mathematically possible in the pass band. Another name for it is flat maximally magnitude filter. The Butterworth type filter was first described by the British engineer Stephen Butterworth. Butterworth solved the equations for two and four pole filters and showed how the latter could be cascaded when separated by vacuum tube amplifiers. This made possible the construction of higher order filters in spite of inductor losses. In 1930, low loss core materials such as molypermalloy had not been discovered and air core audio inductors were rather lossy. Butterworth discovered that it was possible to adjust the component values of the filter to compensate for the winding resistance of the inductors. The frequency response of the Butterworth filter is maximally flat (has no ripples) in the pass band, and rolls off towards zero in the stop band. When viewed on a logarithmic Bode plot, the response slopes off linearly towards negative infinity. For a first-order filter, the response rolls off at −6 dB per octave (−20 dB per decade) (all first-order low pass filters have the same normalized frequency response). For a second-order low pass filter, the response ultimately decreases at −12 dB per octave, a third-order at −18 dB, and so on. Butterworth filters have a

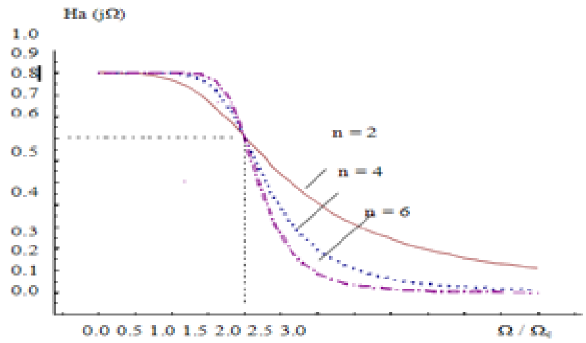

Fig. 2: Frequency Response of the Butterworth filter.

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monotonically changing magnitude function with ω, unlike other filter types that have non-monotonic ripple in the pass band and/or the stop band. Compared with a Chebyshev Type I/Type II filter or an elliptic filter, the Butterworth filter has a slower roll-off, and thus will require a higher order to implement a particular stop band specification. However, Butterworth filters have a more linear phase response in the pass band than the Chebyshev Type I/Type II and elliptic filters. The squared response of the Butterworth filter is given as the function of the cut-off frequency.

$$|Ha(j\Omega)|2 = Ha(j\Omega)\, Ha^*(j\Omega) = \left(\frac{j\Omega}{j\Omega c}\right) 2n \qquad (1)$$

Ωc is the 3db cut off frequency. The first 2n-1 derivative of $|Ha(j\Omega)|2$ at $\Omega=0$ are equal to zero, so the Butterworth response is maximally flat at $\Omega = 0$.

For $\Omega >> \Omega c$, the response of filter is calculated as,

$$|Ha(j\Omega)|2 = \frac{1}{\left(\frac{\Omega}{\Omega c}\right)^{2n}} \;\ldots\ldots\ldots\ldots \qquad (2)$$

The maximum pass band edge attenuation is

$$|Ha(j\Omega p)| = \frac{1}{\sqrt{1+\left(\frac{\Omega p}{\Omega c}\right)^{2n}}} \qquad (3)$$

(Ωp/ Ωc) 2n = ε2 e-jΩt dt] * (4)

So the order of the filter is calculated from the pass edge frequency.

$$n = \log \frac{\log \varepsilon}{\log \Omega p - \log \Omega c} \qquad (5)$$

The minimum stop band edge attenuation is calculated as;

$$|Ha(j\Omega s)|2 = \frac{1}{\left(1+\frac{\Omega s}{\Omega c}\right)^{2n}} \qquad (6)$$

The transfer function Ha (jΩs) of the Butterworth filter is calculated from the order of the filter.

$$Ha^*(j\Omega) = [\int_0^\infty h_a(t) \qquad (7)$$

From the above given expression the filter order and the coefficients are calculated

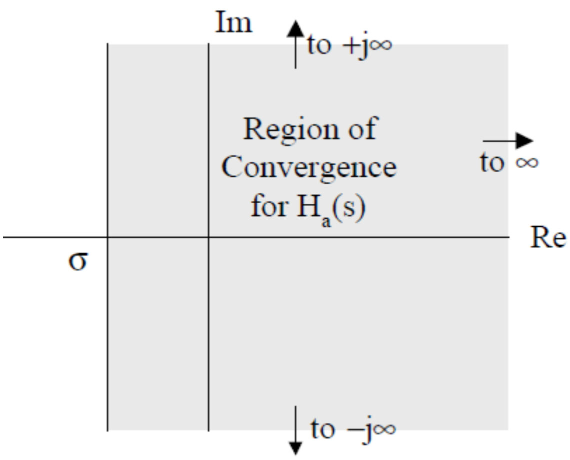


Fig. 3: Region of convergence for the transfer function of the filter.

III. Algorithms for Designing Wavelets to match a specified Signal.

In the paper [3], author presents an algorithms for designing a mother wavelet ψ(x) such that it matches a signal of interest and such that the family of wavelets {2-(j/2)ψ(2-j x-k)} forms an orthonormal Riesz basis of L2(R)are developed. The algorithms are based on a closed form solution for finding the scaling function spectrum from the wavelet spectrum. Many applications of signal representation, adaptive coding and pattern recognition require wavelets that are matched to a signal of interest. Most current design techniques, however, do not design the wavelet directly. They build a composite wavelet from a library of previously designed wavelets, modify the bases in an existing multiresolution analysis or design a scaling function that generates a multiresolution analysis with some desired properties. In this paper [3], two sets of equations are developed that allow us to design the wavelet directly from the signal of interest. Both sets impose band limitedness, resulting in closed form solutions. The first set derives expressions for continuous matched wavelet spectrum amplitudes. The second set of equations provides a direct discrete algorithm for calculating close approximations to the optimal complex wavelet spectrum. The discrete solution for the matched wavelet spectrum amplitude is identical to that of the continuous solution at the sampled frequencies.

IV. Statistically Matched Wavelet Design

The paper [4] presents the design of statistically matched wavelet filter banks that have been extensively studied where it has been formulated as a constrained optimization problem. To assess the coding gain performance of matched filter banks designed according to Vis-&Vis that of the KLT, it is need to extend the parametric analysis. Let Rxx denote the covariance matrix of a process and let the vector, h, of size 2M, consists of the filter coefficients. The matched wavelet that maximizes the energy compaction can be formulated in terms of the objective function J:

J= hTRXX h + λo [1- hTh] + μ1[hTC1 h] + ...μM[hT CMh]......(8)

V. Constructing Wavelets from Desired Signal Functions

In the paper [5], author presents the most applications of orthonormal multiresolution analyses (OMRA) use either Daubechies, Meyer's, or Lemarie's wavelets. However, it would be best if the wavelet matched the signal of interest. The paper [5],presents a technique for generating an OMRA with a wavelet that is matched in the least squares sense to a signal of interest by first developing a method for constructing the scaling function from the wavelet and second, giving the conditions on the wavelet that guarantee an OMRA [6].

VI. Multiresolution Decomposition

Mallat [6] showed that the discrete wavelet transform can be used to generate an orthonormal multiresolution decomposition of a discrete signal consisting of a series of detail functions and a residual low resolution approximation of the original signal. The decomposition is done by convolving the original sequence with a pair of quadrature mirror filters, h (low pass) and g (high pass). In order to perfectly reconstruct the original signal from the detail functions and the residual approximation, the following must be true of the Fourier spectrum magnitudes of h and g.

|H (w) |2 + |G (w) |2 = 1 (9)

Cancellation of any aliasing is guaranteed by setting

gk = (-l)k h1-k (10)

The filters, h and g, are related to the mother wavelet, ψ(x), and the scaling function, φ(x), by their 2-scale relations.

ψ(x)= 2 ∑k gkφ(2x - k) (11)

φ(x)= 2 ∑k hkφ(2x-k) (12)

or in the frequency domain by

ψ(w)=G(w/2) φ(w/2) and φ(w)=H(w/2) φ(w/2) (13)

A. Constructing φ from ψ

A recursive equation for finding φ(w) from ψ(w) can be found by taking the magnitude squared of φ(w) adding them and substituting equation in

$$|H(w)|2 + |G(w)|2 = 1 \quad (14)$$

Giving

$$|\varphi(2w)|2 + |\psi(2w)|2 = |\varphi(w)|2 \quad (15)$$

Substituting w = Πk, then w = nk/2 and so on, leads to the following closed form solution. So, given any known wavelet its corresponding scaling function can be found directly

VII. Flow Chart for Signal denoising.

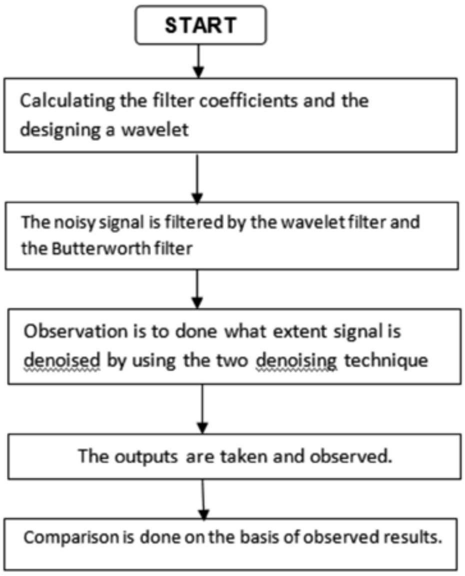


Fig. 3: Flow chart for the signal denoisin

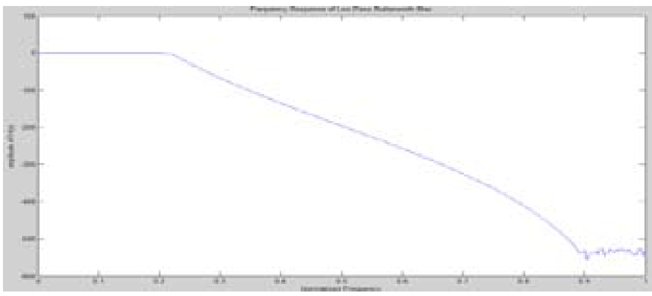
VIII. Design steps

A. Algorithm for the wavelet designing from IIR filter coefficients

1. Set the initial pass band and stop band edge frequencies.
2. Set the length of Butterworth window, N = 22
3. Calculating the passband attenuation

$$|\,Ha(j\Omega p)\,| = \frac{1}{\sqrt{1+\left(\frac{\Omega p}{\Omega c}\right)}}2n$$

4 Get plot of low . Calculating the stopband attenuation



$$|\,Ha(j\Omega s)\,|^2 = \frac{1}{\left(1+\frac{\Omega s}{\Omega c}\right)}2n$$

5 Applying Matlab command to obtain Butterworth filter coefficients.[b,a]=butter(N,wn);pass utterworth filter.

B. Designing Wavelet Scaling function from filter coefficients

5. Filter coefficients are input to scaling function, i.e., b= $g_0 g_0$
6. Getting plot of scaling function.

Getting wavelet coefficients using relation

$$a_k = (-1)^k \, g_{N-1-K}$$

7. Getting plot of wavelet function.

C. Filtering noise through the Butterworth filter and scaling function.

8. Applying the noisy signal as an input on the Butterworth filter.
9. Level of filtering is chosen.
10. Noisy signal is applied on the scaling function.

IX. Results

Generation of the clean signal and then some noise is added

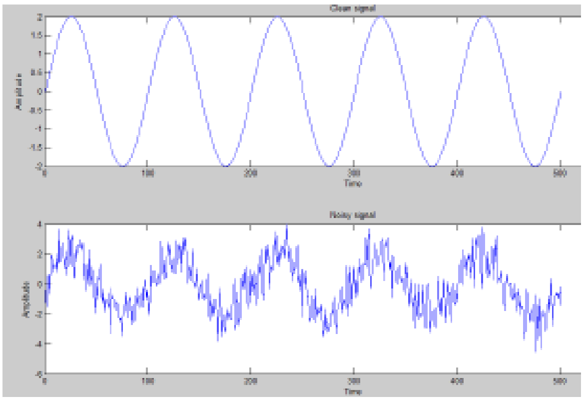


Fig. 4: Clean and noisy signal. Butterworth filter response

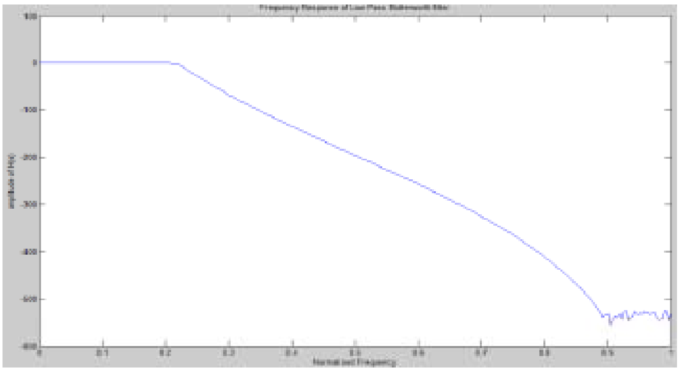


Fig. 5: Frequency response of LPF Butterworth filter.

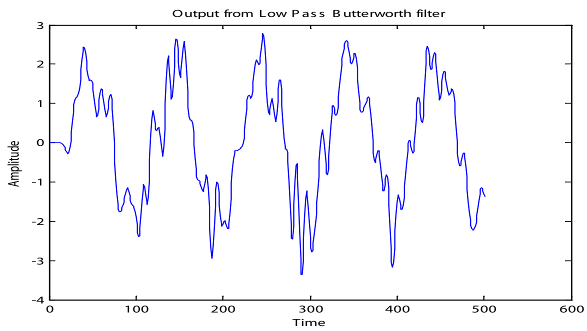


Fig. 6: Output of Butterworth filter for noisy signal

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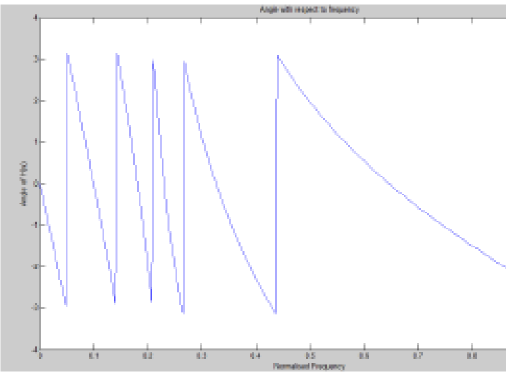


Fig. 7: Phase response of Butterworth filter

Designing of the matched wavelet by using the Butterworth filter coefficients

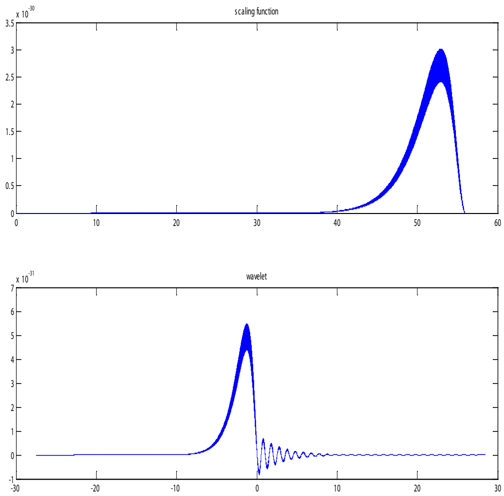


Fig. 8: Wavelet and scaling function

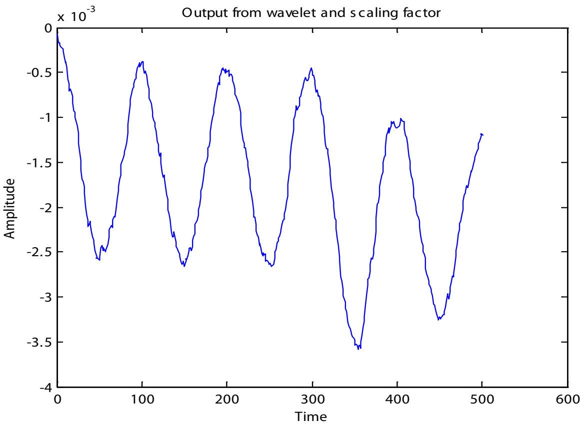
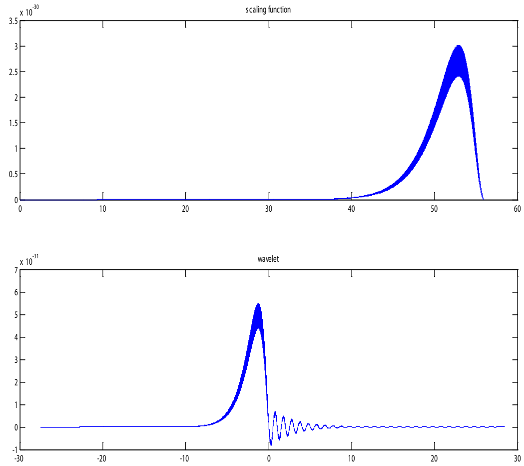


Fig. 9: Matched wavelet output for signal denoising



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