Realization of Flat-to-Negative Dispersion of a Helical Slow-Wave Structure for a Wideband Travelling-Wave Tube using T-Shaped Dielectric Helix-Support Rods

1Sumit K Varshney, 1Amit K Varshney, 2Raktim Guha, 2Sanjay K Ghosh

1Dept. of ECE, SKFGI, Mankudu, West Bengal, India
2Central Electronics Engg. Research Institute (CEERI), Pilani (CSIR), West Bengal, India

Abstract
Field analysis of the helical structure supported by identical dielectric T-shaped rods symmetrically arranged around the helix, the whole enclosed in an overall metal envelope, has been carried out in the sheath-helix model. The analysis established the superiority of the T-shaped supports over their rectangular- and inverted-T-shaped counterparts with respect to providing flat- to-negative dispersion characteristics as required for wideband travelling-wave tubes for electronic warfare applications.

Keywords
Wideband TWTs, Helical Slow-Wave Structure, Inhomogeneous Helix Loading, Electronic Warfare

I. Introduction
Broadbanding a helix TWT continues to be an important consideration for the development of ECM and ECCM TWTs. The method of broadbanding the device depends on the control of helix dispersion which can be realized by loading the helix by anisotropic envelope [1]-[3], by tapered geometry dielectric helix supports [4] and using multi-dispersion structures. Although rectangular or circular dielectric helix supports inhomogeneously load the helix [5]-[9] the inhomogeneous loading provided by them, contrary to wedge-shaped supports, does not yield the desired flat-to-negative dispersion for broadband TWTs at a high interaction impedance value as necessary for a broadband TWT. The present work attempts to study the potential of T-shaped dielectric helix supports with the help of simple field analysis of the structure in the sheath-helix model [1-2].

II. Field Analysis
Let us consider the structure consisting of a helix supported by three identical dielectric T-shaped support rods symmetrically arranged around the helix, the whole enclosed in an overall metal envelope (Fig. 1). The structure is analyzed in the sheath-helix model replacing the actual structure by an equivalent structure consisting of four tube regions of effective relative permittivity values \( \varepsilon_{r_{p}} \) where the subscripts \( P = 1,2,3,4 \) refer to the free-space region inside the helix (\( \varepsilon_{r_{1}} = 1 \)) [10-11], the free-space gap, to take into account the effect of finite helix thickness, between the mean helix radius \( a \) (being equal to the sheath-helix radius) and the beginning of the dielectric region (\( \varepsilon_{r_{2}} = 1 \)), i.e., at the outer radius \( a_{0} \) of the helix, and the two equivalent continuous dielectric regions of relative permittivities \( \varepsilon_{r_{3}} \) and \( \varepsilon_{r_{4}} \), respectively, in which the vertical and horizontal limbs of the T-shaped supports are smoothed out (Fig.1).

The electric and magnetic fields in the different regions of the structure are given by [1-2]:

\[ E_{0,p} = -\left( j \omega \mu / \gamma \right) \left[ C_{p} I_{r} - D_{p} K_{r} \right] \]

\[ H_{0,p} = \left( j \omega \varepsilon / \gamma \right) \left[ A_{p} I_{r} + B_{p} K_{r} \right] \]

\[ E_{Z,p} = A_{p} I_{or} + B_{p} K_{or} \]

\[ H_{Z,p} = C_{p} I_{or} + D_{p} K_{or} \]

where \( I_{or} = I_{or}(\gamma) \); \( K_{or} = K_{or}(\gamma) (\gamma = 1,2) \); \( \gamma \) being the radial propagation constant. The subscript \( p \) refers to the region of the structure equivalent (Fig. 1). A, B, C and D are the field constants. \( I_{0} \) and \( K_{0} \) are the zeroth order modified Bessel functions of the first and second kinds, respectively; and \( I_{1} \) and \( K_{1} \) are the corresponding first order modified Bessel functions.

Fig. 1: Helix With Discrete Dielectric Supports: (a) Rectangular-Shaped (b) T-shaped (c) Inverted-T-shaped cross sections and their equivalent structure consisting of a helix surrounded by continuous free-space and dielectric tube regions.
Further, here, \( B_2 = D_2 = 0 \) in order to prevent the fields from blowing to infinity at the axis of the helix \([1-2]\).

Hence, with the help of field expressions (1)-(4), normalized wave impedance functions of the structure may be written as \([12]\)

\[
Z_{E,p} = \frac{jE_{z,p}}{\eta_o H_{\phi,p}} = \frac{\gamma}{k} \left[ I_{1a} + \left( \frac{B_2}{A_2} \right) K_{1a} \right] \left[ I_{1a} + \left( \frac{B_2}{A_2} \right) K_{1a} \right]
\]

(5)

\[
Z_{H,p} = \frac{jE_{q,p}}{\eta_o H_{\phi,p}} = \frac{k}{\gamma} \left[ I_{1a} - \left( \frac{D_2}{C_2} \right) K_{1a} \right] \left[ I_{1a} + \left( \frac{D_2}{C_2} \right) K_{1a} \right]
\]

(6)

The boundary conditions used are:

(a) \( r = a \) (sheath-helix radius):

\[
H_{Z,1} \sin \psi + H_{Z,2} \cos \psi = H_{Z,2} \sin \psi + H_{Z,2} \cos \psi
\]

(7)

\[
E_{Z,1} \sin \psi + E_{Z,1} \cos \psi = 0
\]

(8)

\[
E_{Z,2} \sin \psi + E_{Z,2} \cos \psi = 0
\]

(9)

\[
E_{Z,1} = E_{z,2}
\]

(10)

(b) \( r = b \) (metal envelope radius):

\[
E_{a,4} = 0; E_{z,4} = 0
\]

(11)

where \( \psi = \cot^{-1}(2\alpha/a) \) is the helix pitch angle, From (7), using (5), (6), (8), (9) and (10), we may write at the sheath-helix radius

\[
\frac{1}{Z_{H,1}} + \frac{1}{Z_{E,1}} \cot^2 \psi = \frac{1}{Z_{H,2}} + \frac{1}{Z_{E,2}} \cot^2 \psi
\]

(12)

From (12), using (5) and (6), we obtain

\[
k^2 \cot^2 \psi = \frac{\left[ I_{1a} + \left( \frac{D_2}{C_2} \right) K_{1a} \right] \left[ I_{1a} - \left( \frac{D_2}{C_2} \right) K_{1a} \right]}{\left[ I_{1a} + \left( \frac{B_2}{A_2} \right) K_{1a} \right] \left[ I_{1a} + \left( \frac{B_2}{A_2} \right) K_{1a} \right]}
\]

(13)

The ratios of constants \( B_2/A_2 \) and \( D_2/C_2 \) refer to the free-space region outside the helix. The ratios \( B_2/A_4 \) and \( D_2/C_4 \) may be expressed, with the help of the boundary condition \( E_{a,4} = 0 \) and \( E_{z,4} = 0 \), respectively (see (11)), as

\[
\frac{B_2}{A_4} = -\frac{I_{ob}}{K_{ob}} \quad \text{(a)}
\]

\[
\frac{D_2}{C_4} = \frac{I_{1b}}{K_{1b}} \quad \text{(b)}
\]

(14)

Now, we can transfer the ratio (14) step by step inward with the help of the following impedance boundary conditions at the interface between the two equivalent structure regions \( r = a_i \) (Fig. 1). For this purpose, we may use the following impedance boundary condition at the interface between the \( p \)th and \( (p-1) \)th equivalent structure regions:

\[
Z_{E,p} = Z_{E,p-1}
\]

(15)

\[
Z_{H,p} = Z_{H,p-1}
\]

(16)

Thus, with the help of (14(a)) and (15) we obtain \( B_2/A_2 \) and \( B_2/A_3 \) as follows:

\[
\frac{B_2}{A_2} = \frac{z_{EE}^2}{B_3} \frac{I_{oa}}{I_{1a}} \frac{z_{EE}^2}{B_3} \frac{I_{oa}}{I_{1a}} + \frac{z_{EE}^2}{B_3} \frac{K_{oa} I_{1a}}{K_{oa} I_{1a}}
\]

(17)

\[
\frac{B_2}{A_3} = \frac{z_{EE}^2}{B_3} \frac{I_{oa}}{I_{1a}} + \frac{z_{EE}^2}{B_3} \frac{K_{oa} I_{1a}}{K_{oa} I_{1a}} + \frac{z_{EE}^2}{B_3} \frac{B_4}{A_4} \frac{K_{oa} I_{1a}}{K_{oa} I_{1a}}
\]

(18)

and

\[
\frac{B_2}{A_3} = \frac{z_{EE}^2}{B_3} \frac{I_{oa}}{I_{1a}} + \frac{z_{EE}^2}{B_3} \frac{B_4}{A_4} \frac{K_{oa} K_{1a}}{K_{oa} K_{1a}}
\]

where \( a_0 \) is the outer radius of the helix and \( a_1 \) is defined following (14) (Fig. 1).

III. Results and Discussion

Let us consider an equivalent structure taking typically \( b/a = 2 \) and helix thickness to mean helix radius \( 2(a_0 - a)/a \). The dispersion relation (13) has been used to plot the dispersion characteristics: \( k \cot \psi \) versus \( k_{a} \cot \psi \) using MATLAB. The structure represents a helix supported by three identical discrete dielectric rods, for instance, of rectangular cross section (Fig. 2) and of T-shaped (Fig. 3) cross sections. For the single dielectric equivalent structure, by increasing the value of the effective relative permittivity \( \varepsilon' \), the dispersion characteristic becomes flatter, and even slightly negative, however, at the cost of the RF phase velocity, which calls for a reduced synchronous beam velocity and a correspondingly a reduced beam voltage amounting to a reduced device RF output power to be derived from the beam power. Interestingly, for T-shaped dielectric helix-support rods, the structure can be optimized for the desired flat-to-negative dispersion at a relatively higher RF phase velocity value and hence, correspondingly, for a higher beam velocity/voltage (Fig. 3(a)), unlike for an inverted T-shaped support rods, for which the structure cannot be so optimized (Fig. 3(b)). Thus, we have obtained the typical optimized values as \( \varepsilon'_c = 1.5 \) and \( \varepsilon'_e = 3.5 \) for \( b/a = 2 \) (normalized envelope radius) and \( 2(a_0 - a)/a = 0.1 \) (normalized helix thickness to mean helix radius). Choosing the T-shaped dielectric supports to be made of thermally conducting APBN (\( \varepsilon_r = 5.1 \)), from geometrical considerations, the dimensions of the discrete supports turn out to be 2.0433 mm \( \times \) 0.8 mm (width \( \times \) height) for the upper limb and 0.2873 mm \( \times \) 0.15 mm for the lower limb of T.
The dispersion characteristics obtained analytically have been validated against CST-MW Studio simulation (Fig. 4) showing the agreement of the present analysis with the simulation, more so at lower frequencies. It is felt that the agreement would be closer if one would resort to the tape-helix model instead of the present sheath-helix model [2]. The analytical results can be further refined by smoothing out the discrete T-shaped supports by a number of equivalent continuous dielectric tube regions, with due consideration to increasing the number of regions for convergent results, instead of the two regions considered in the present analysis for the sake of simplicity.

IV. Conclusion

The results have definitely shown that one should prefer T-shaped dielectric helix-support rods to conventional rectangular support rods for the desired flat-to-negative dispersion, which is an important consideration in the design of a broadband TWT for electronic warfare application.

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References


