Preliminary Studies on Injection Locking of Oscillators

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Abstract
Injection locking characteristics of oscillators are reviewed both theoretically and experimentally. Theoretical results coupled with experimental findings are presented. A simple method of deriving the equation for locking range is reported.

Keywords
Injection Locking, Locking Range Equation, Nonlinearity

I. Introduction
A self oscillating system or a self-oscillator is a primary source of oscillations operating under self-exciting conditions. Any oscillator is a nonlinear system converting the dc power of a source into ac energy. At the moment of starting the self-oscillator, free oscillations are generated in its oscillatory system due to switching on the power source, breaking the circuits, electric fluctuations etc. The positive feedback amplifies these initial conditions, in which case during the initial stage of the process, when the amplitudes are small, we may consider the system to be linear as the amplification is practically linear. From energy considerations, the process which increases the amplitudes is explained by the fact that the amount of energy supplied by the amplifier to the inertia load network is greater than the amount of energy dissipated in the system. The nonlinearity of the system (curvature of the current-voltage characteristic of the amplifying element) begins to manifest itself with the rise of the amplitudes, and consequently the gain of the system drops. The amplitudes stop increasing when the amplification is reduced to the level at which the damping of the oscillations in the load circuit is balanced; in this case the energy supplied by the amplifier per period proves to be equal to the energy consumed in the circuit load during the same time.

An oscillator operating under steady-state conditions is a conventional nonlinear amplifier excited by the oscillations produced in the oscillator itself; these oscillations are fed from the oscillatory system of the amplifier and applied to its input through a feedback loop. When the amplitude and phase of oscillations satisfy certain conditions, the behavior of a self-oscillator is identical to that of a separately excited oscillator. In the last stage of the transient conditions the behavior of the oscillator is determined mainly by the nonlinear character of the system, so that the steady state of the oscillator cannot be described without taking the nonlinearity into account.

Injection locking becomes useful in a number of applications including frequency division [1], quadrature generation [2], [3] and oscillators with finer phase separations [4]. However, injection pulling on the other hand typically proves undesirable. For example, in a broadband transceiver [5], the two voltage controlled oscillators may pull each other as a result of substrate coupling.

In this paper, we have derived the locking equation for an injection synchronized oscillator using two approaches. The first method is based on simple phasor algebra and the second one is derived using the conventional circuit theory approach. The locking range of such an oscillator is derived and experimental results showing the variation of frequency detuning with injection amplitude and locking range have been presented. The circuit shown in Fig. 1 shows an injection synchronized transistorized oscillator. The tank operates at the resonance frequency \( \omega_0 = \frac{1}{\sqrt{LC}} \), thus contributing no phase shift at this frequency and the ideal inverting buffer follows the transistor to create a total phase shift of 360° around the feedback loop. Now when an additional phase shift \( \Phi_0 \) is inserted in the loop, the circuit can no longer oscillate at \( \omega_0 \), because the total phase shift at this frequency deviates from 360° by \( \Phi_0 \). Hence the oscillation frequency must change itself to a new value \( \omega_1 \) such that the tank contributes enough phase shift to cancel the effect of \( \Phi_0 \). In Fig. 1, the additional phase shift has been realized by adding a sinusoidal injection current \( I_{inj} \) to the collector of the transistor ‘T’. If the amplitude and frequency of \( I_{inj} \) are chosen properly, then the circuit will oscillate at \( \omega_{inj} \) rather than at \( \omega_0 \), and injection locking occurs.

II. Derivation of the Locking Equation
Locking range refers to the range of frequencies of \( \omega_{inj} \) across which injection locking holds. To match the increasingly greater phase shift introduced by the tank, the angle between the \( I_{osc} \) and \( I_T \) must also increase, requiring that \( I_{osc} \) will rotate anticlockwise. Using trigonometric identity, it is not difficult to show that
Again,
\[
\sin \phi_0 = \frac{I_{\text{inj}}}{I_T} \sin \theta \\
I_{\text{arc}} + I_{\text{inj}} \cos \theta
\]
\[
\Rightarrow \tan \phi_0 = \frac{I_{\text{inj}} \sin \theta}{I_{\text{arc}} + I_{\text{inj}} \cos \theta}
\]

A second-order parallel tank circuit consisting of 'L', 'C' and 'R' exhibits an impedance [7] of

\[
Z(j\omega) = \frac{1}{R + j\left(\omega C - \frac{1}{\omega L}\right)} = \frac{1 - j\left(\omega C - \frac{1}{\omega L}\right)}{R^2 + \left(\omega C - \frac{1}{\omega L}\right)^2}
\]

Thus the phase shift introduced by the tank circuit near resonance is given by

\[
\phi = -\tan^{-1}\left[\frac{\omega C - \frac{1}{\omega L}}{R}\right] = -\tan^{-1}\left[\frac{Q}{\omega_0^2 (\omega^2 - \omega_0^2)}\right] \approx -\tan^{-1}\left[\frac{2Q}{\omega_0} (\omega - \omega_0)\right]
\]

where the following simplifications have been used:

\[
\omega_0^2 - \omega^2 \approx 2\omega_0 (\omega_0 - \omega) ; \quad Q = \frac{R}{\omega L} \quad \text{and} \quad \omega_0 = \frac{1}{\sqrt{LC}}
\]

Hence equating (2) with the phase shift introduced by the circuit, one gets

\[
2Q \omega_0 \left[ (\omega - \omega_T) - (\omega_0 - \omega_0) \right] = -\frac{I_{\text{inj}} \sin \theta}{I_{\text{arc}} + I_{\text{inj}} \cos \theta}
\]

\[
2Q \omega_0 \left[ \frac{d\theta}{dt} - \Delta \omega_0 \right] = -\frac{I_{\text{inj}} \sin \theta}{I_{\text{arc}} + I_{\text{inj}} \cos \theta}
\]

\[
\frac{d\theta}{dt} = \Delta \omega_0 - \frac{\omega_0 \left( \frac{I_{\text{inj}}}{I_{\text{arc}}} \right) \sin \theta}{2Q \left( \frac{I_{\text{inj}}}{I_{\text{arc}}} \right)} \cos \theta
\]

Since \( \frac{I_{\text{inj}}}{I_{\text{arc}}} \neq 1 \), (3) reduces to

\[
\frac{d\theta}{dt} = \Delta \omega_0 - \frac{\omega_0 \left( \frac{I_{\text{inj}}}{I_{\text{arc}}} \right) \sin \theta}{2Q \left( \frac{I_{\text{inj}}}{I_{\text{arc}}} \right)} \cos \theta
\]

where '\( \Delta \omega_0 \) ' is open-loop frequency error and (4) is the famous Adler’s equation [6].

### III. Oscillator Under Weak Injection

In the following analysis, we take a negative differential conductance oscillator in presence of an injection signal \( I_{\text{inj}} (t) \). Here, \( R' \) accounts for the losses in the tank circuit and we model the oscillator as a one-port circuit consisting of a parallel tank circuit and a non-linear element \( G_{\text{NL}} \). Since a linear oscillator does not injection lock, the non-linearity in the circuit will aid the process of injection locking. Typically the non-linearity in Fig. 3 arises because of the non-linearities present in the active core of the transistor [8] in Fig.1.

Application of Kirchhoff’s current law, one gets

\[
\frac{dv_0}{dt} = \frac{I_{\text{inj}}}{C} - \frac{1}{LC} \int v_0 dt - \frac{v_0}{RC} + \frac{G_{\text{NL}}}{C} v_0
\]

\[
C \frac{d^2 v_0}{dt^2} + \frac{v_0}{L} = \left[ G_{\text{NL}} - \frac{1}{R} \right] \frac{dv_0}{dt} = \frac{dI_{\text{inj}}}{dt}
\]

The output of the oscillator and the synchronizing signal are taken as

\[ I_{\text{inj}} = I_{\text{inj}} e^{j\omega_{\text{inj}}t} \quad \text{and} \quad v_0 = V(t) e^{j(\omega_{\text{inj}} + \theta(t))} \] ; where \( V(t) \) is the envelope of the oscillator output and ' \( \theta(t) \) ' is the output phase modulation because of the synchronizing signal.

\[
\therefore \frac{dv_0}{dt} = e^{j(\omega_{\text{inj}} + \theta(t))} \left[ \frac{dV}{dt} + j \left( \omega_{\text{inj}} + \frac{d\theta}{dt} \right) V(t) \right]
\]

\[
\frac{d^2 v_0}{dt^2} = \left[ \frac{d^2 V}{dt^2} + j \left( \omega_{\text{inj}} + \frac{d\theta}{dt} \right) \frac{dV}{dt} + j \left( \frac{d\theta}{dt} \right)^2 V(t) \right] + e^{j(\omega_{\text{inj}} + \theta(t))}
\]

and

\[
\frac{dI_{\text{inj}}}{dt} = j \omega_{\text{inj}} I_{\text{inj}} e^{j(\omega_{\text{inj}} + \theta)} = j \omega_{\text{inj}} I_{\text{inj}} e^{j\omega_{\text{inj}} + \theta}
\]

Using these results in (5), one gets
Equating the real and imaginary parts, it is not difficult to show that

\[
C \left[ \frac{d^2 V}{dt^2} + \frac{j d^2 \theta}{dt^2} V(t) + 2 j \left( \frac{\omega_{o_2}}{dt} + \frac{d \theta}{dt} \right) \frac{dV}{dt} \right] \quad (6)
\]

In the steady state, \( \frac{d \theta}{dt} = 0 \), \( \frac{dV}{dt} = 0 \), i.e.,

\[
(\omega_o, - \omega_{o_2}) - \omega_{o_2} \frac{d \theta}{dt} = \frac{\omega_{o_2} I_0}{2IQ_{o_2}} \sin \theta
\]

the ideal locking or synchronizing range of the oscillator.

**IV. Results and Discussions**

The oscillator output phase variation and its steady state value is shown in Fig. 5. The result is obtained by solving (9). The free running frequency of the oscillator is 45 Hz and is shown in Fig. 6. The experimental validation is done with the help of MATLAB SIMULINK and is shown in Fig. 4. In Fig. 7 the injection signal frequency is varied (hence the frequency detuning) and the corresponding injection signal amplitude is noted at the verge of synchronization, shown in table-1. Finally, Fig. 8 shows the variation of locking range with the frequency detuning when the injection signal amplitude is kept fixed at 150.
Table 1: Experimental Data for Frequency Detuning With Injection Amplitude

<table>
<thead>
<tr>
<th>Frequency detuning in Hz</th>
<th>Injection Amplitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>25</td>
</tr>
<tr>
<td>3</td>
<td>73</td>
</tr>
<tr>
<td>5</td>
<td>117</td>
</tr>
<tr>
<td>7</td>
<td>165</td>
</tr>
</tbody>
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V. Acknowledgement
The authors are thankful to the management of Central Institute of Technology, Assam, India for giving an opportunity for carrying out this work.

References


